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Fermi's Two Golden Rules

WEAK INTERACTION (1)

Presentation based on "*Introduction to Elementary Particles*"
by David Griffiths

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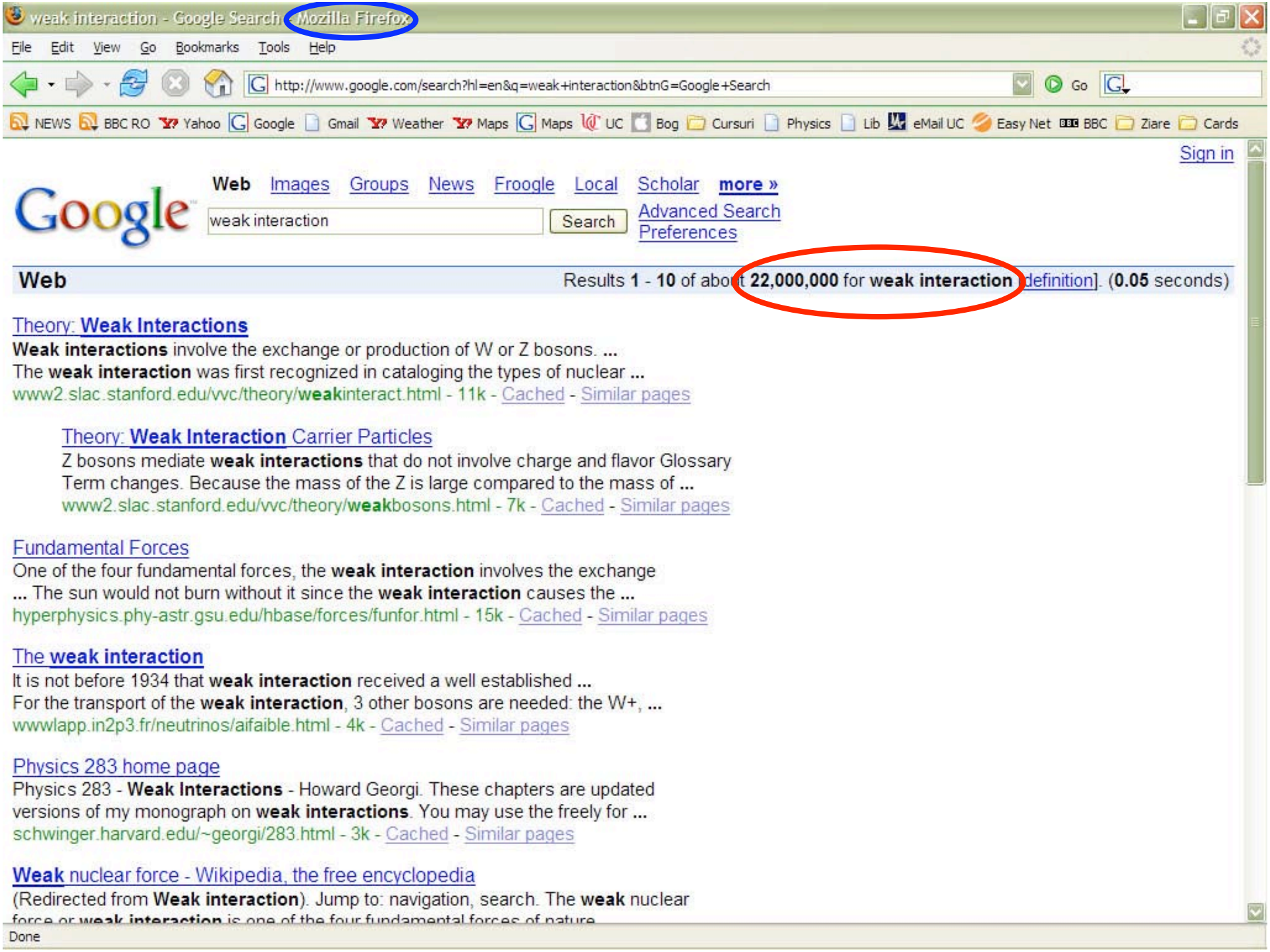
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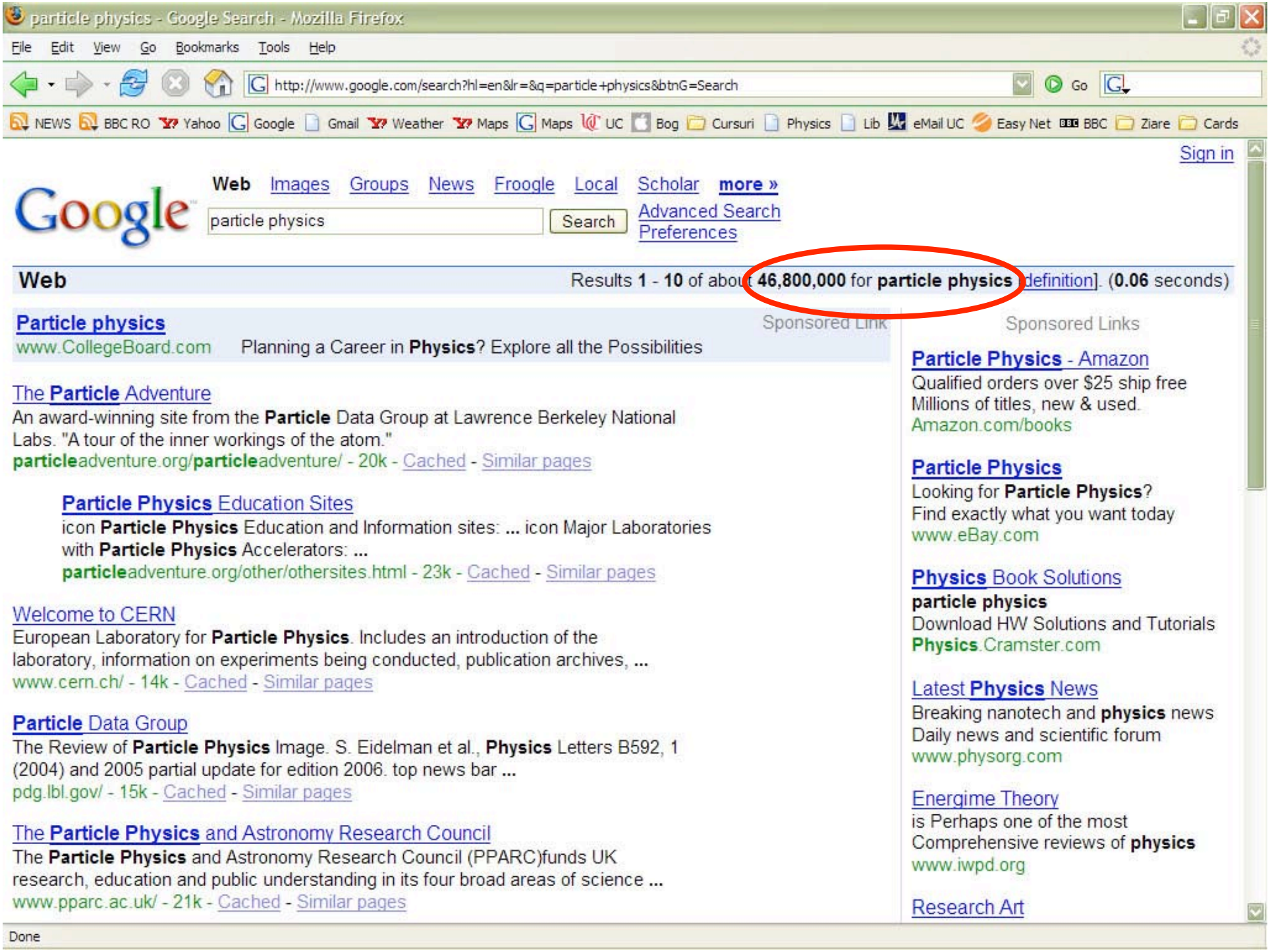
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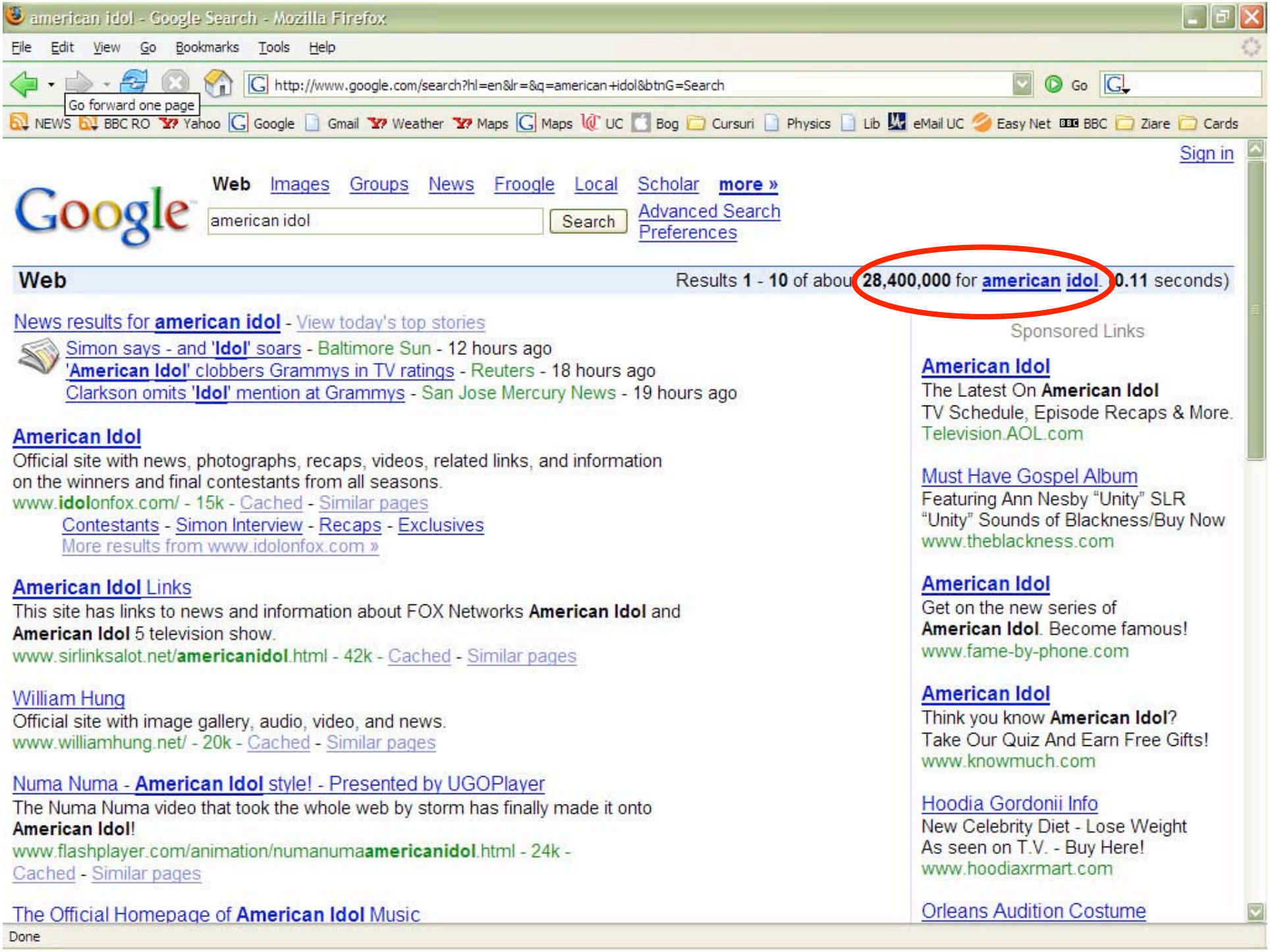
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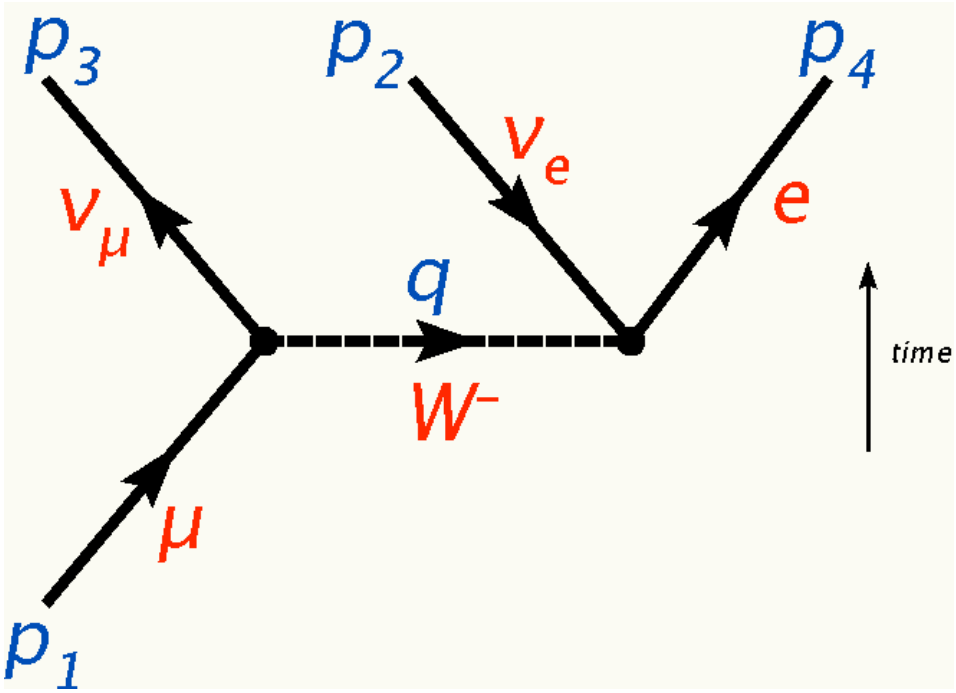

$$\mu \rightarrow e + \nu_\mu + \bar{\nu}_e$$

The amplitude :

$$\mathcal{M} = \frac{g_W^2}{8(M_W c)^2}\left[\bar{u}(3)\gamma^\mu\left(1-\gamma^5\right)u(1)\right]\left[\bar{u}(4)\gamma_\mu\left(1-\gamma^5\right)v(2)\right]$$

As before : $$\left\langle|\mathcal{M}|^2\right\rangle = 2\left(\frac{g_W}{M_W c}\right)^4 (p_1 \bullet p_2)(p_3 \bullet p_4)$$

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(35.12) over k and dividing by t_0:

$$w = \frac{1}{t_0} \int |a_k^{(1)}(t \geq t_0)|^2 \rho(k)\, dE_k \tag{35.13}$$

where $\rho(k)\, dE_k$ is the number of final states with energies between E_k and $E_k + dE_k$. The concept of an energy density $\rho(k)$ of final states is sensible, since we are considering the case in which the transition is to one or another of a continuous set of dissociated states. We now take advantage of the fact that the breadth of the main peak in Fig. 31 becomes small as t_0 becomes large, and we regard $\langle k|H'|m\rangle$ and $\rho(k)$ as quantities sufficiently independent of E_k so that they can be taken outside the integral in Eq. (35.13). We further simplify the integral by changing the integration variable from E_k to $x \equiv \frac{1}{2}(\omega_{km} - \omega)t_0$ and extending the limits on x to $\pm\infty$. Substitution of (35.12) into (35.13) then gives[1]

$$w = \frac{2\pi}{\hbar} \rho(k) |\langle k|H'|m\rangle|^2 \tag{35.14}$$

where we have made use of the result $\int_{-\infty}^{\infty} x^{-2} \sin^2 x\, dx = \pi$. This expression for w is independent of t_0, as expected.

There may be several different groups of final states $k_1, k_2, \ldots$, all of which have about the same energy $E_m + \hbar\omega$ but for which the perturbation matrix elements $\langle k_i|H'|m\rangle$ and the densities of states $\rho(k_i)$, although nearly constant within each group, differ from one group to another. Then Eq. (35.14), with k replaced by k_i, gives the transition probability per unit time to the ith group.

It is apparent that the foregoing treatment fails to give a transition probability that is proportional to the time if the final as well as the initial state is discrete. In this case, Eq. (35.12) shows that $|a_k^{(1)}(t \geq t_0)|^2$ depends in a peculiar way on t_0 and on $\omega_{km} - \omega$. We return to this situation in Chap. 11 in connection with radiation processes.

IONIZATION OF A HYDROGEN ATOM

As an example of the first-order time-dependent perturbation theory, we now calculate the probability of ionization of a hydrogen atom initially in its ground state when it is placed in a harmonically time-varying electric field. We might, for instance, think of the atom as being placed between the plates of a capacitor to which an alternating voltage is applied. This is, of course, not a realistic situation, since the circular

[1] Equation (35.14) together with its analog for $\omega = 0$, which will be discussed in Sec. 37, is so useful that it was called "Golden Rule No. 2" by E. Fermi, "Nuclear Physics," p. 142 (University of Chicago Press, Chicago, 1950).

range of direction of the momentum vector as well as of its magnitude. We write the number of these states as $\rho(\beta)\, dE_\beta$, where dE_β is the energy range and $\rho(\beta)$ is evidently a differential in the other parameters. Then the total probability of finding the system in one or another of this group of final states is a summation of $|\langle\beta|(S-1)|\alpha\rangle|^2$:

$$\int |\langle\beta|(S-1)|\alpha\rangle|^2 \rho(\beta)\, dE_\beta = \hbar^{-2} \int |\langle\beta|T|\alpha\rangle|^2 \left| \int_{-\infty}^{\infty} g(t) e^{i\omega_{\beta\alpha} t}\, dt \right|^2 \rho(\beta) \hbar\, d\omega_{\beta\alpha}$$

where we have replaced dE_β by $\hbar\, d\omega_{\beta\alpha}$ since ω_α is constant. Now the Fourier transform of $g(t)$ is strongly peaked at $\omega_{\beta\alpha} = 0$, so that we can remove $\rho(\beta)|\langle\beta|T|\alpha\rangle|^2$ from inside the integral over $\omega_{\beta\alpha}$ and extend the limits to $\pm\infty$. Then

$$\int_{-\infty}^{\infty} \left| \int_{-\infty}^{\infty} g(t) e^{i\omega_{\beta\alpha} t}\, dt \right|^2 d\omega_{\beta\alpha} = 2\pi \int_{-\infty}^{\infty} |g(t)|^2\, dt$$

and this is essentially equal to $2\pi t_0$ if $g(t)$ has the form shown in Fig. 34. The transition probability per unit time is then given by

$$w \equiv \frac{1}{t_0} \int |\langle\beta|(S-1)|\alpha\rangle|^2 \rho(\beta)\, dE_\beta = \frac{2\pi}{\hbar} \rho(\beta) |\langle\beta|T|\alpha\rangle|^2 \tag{37.3}$$

Equation (37.3) is exact. Comparison of the second term on the right side of Eq. (36.35) with the corresponding term of (36.36) shows that a perturbation approximation to (37.3) is obtained by replacing χ_α^+ by u_α in (37.2). This replacement gives Fermi's "Golden Rule No. 2."[1]

SCATTERING CROSS SECTION

An expression for the differential scattering cross section is most conveniently obtained from Eq. (37.3) by using box normalization. We choose the u_β to be the momentum eigenfunctions

$$u_\beta(\mathbf{r}) = L^{-\frac{3}{2}} \exp(i\mathbf{k}_\beta \cdot \mathbf{r})$$

in which case $\rho(\beta)$ is given by Eq. (35.18):

$$\rho(\beta) = \frac{\mu L^3}{8\pi^3 \hbar^2} k_\beta\, d\Omega_\beta \tag{37.4}$$

where $d\Omega_\beta$ is the infinitesimal element of solid angle associated with the direction of $\mathbf{k}_\beta$.

The value of w obtained by substitution of (37.4) into (37.3) is the number of scatterings into $d\Omega_\beta$ per unit time when there is initially one system in the volume L^3. This is an incident flux of v_α/L^3 per unit area

[1] E. Fermi, "Nuclear Physics," p. 142 (University of Chicago Press, Chicago, 1950); see also Eq. (35.14).

The result of the calculation is that the cross section is very small unless the momentum vector of the incident electron is nearly parallel to the line that joins the two nuclei, and also unless the initial and final electron momenta are nearly parallel. These three directions can have an angular spread in radians that is of the order of the ratio of the wavelength of the electron to the size of the atom. This is analogous to the result obtained in the preceding subsection for the inelastic collision of a fast electron with a hydrogen atom: The angular spread of the scattered electron was found to be roughly $1/k_\alpha a_0$. It is also in agreement with the wave-packet description of the process, since a localization of the electron by an atomic size a in a direction transverse to its motion produces an uncertainty in the transverse-momentum component of amount $\hbar/a$ and an angular spread of order $\hbar/ap \approx 1/ka$.

SECOND-ORDER PERTURBATION THEORY

The first-order perturbation or Born approximation for the T matrix element (38.21) was obtained by replacing $\chi_{\alpha a}{}^+$ by $u_{\alpha a}$. In similar fashion, the second-order approximation is obtained by replacing $\chi_{\alpha a}{}^+$ by the second term of the perturbation series that is analogous to Eq. (37.15), to obtain

$$\frac{1}{\hbar}\iiiint u^*_{\beta b}(\mathbf{r}'_1,\mathbf{r}'_2)H'(\mathbf{r}'_1,\mathbf{r}'_2)G^+_{0\omega\alpha a}(\mathbf{r}'_1,\mathbf{r}'_2;\mathbf{r}_1,\mathbf{r}_2)H'(\mathbf{r}_1,\mathbf{r}_2)$$
$$u_{\alpha a}(\mathbf{r}_1,\mathbf{r}_2)\, d^3r'_1\, d^3r'_2\, d^3r_1\, d^3r_2$$

Substitution for the propagator can be made as in the first line of Eq. (38.20), where now the summations are over γ and c rather than over β and b. We thus obtain for the second-order contribution to the T matrix element[1]

$$\mathsf{S}_\gamma \mathsf{S}_c (E_{\alpha a} - E_{\gamma c} + i\epsilon)^{-1} \langle \beta b|H'|\gamma c\rangle\langle\gamma c|H'|\alpha a\rangle \tag{38.30}$$

This expression may be thought of as describing a two-step process in which the system makes a transition from the initial state αa to all possible intermediate states γc under the influence of the perturbation H', and then a similar transition from γc to the final state βb. Energy is conserved between initial and final states but need not be for the intermediate states. These have only a transient existence, and according to the uncertainty relation (3.3) it is impossible to determine the energy of such short-lived states with any precision. Thus it is not surprising that their contributions to the second-order T matrix element are inversely proportional to this energy discrepancy. Similar second-

[1] This was called "Golden Rule No. 1" by E. Fermi, "Nuclear Physics," p. 148 (University of Chicago Press, Chicago, 1950).

Claude Cohen-Tannoudji
Bernard Diu
Franck Laloë

QUANTUM MECHANICS

Volume II

Translated from the French by **Susan Reid Hemley, Nicole Ostrowsky, Dan Ostrowsky**

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The system is described at time t by the normalized ket $|\psi(t)\rangle$. We want to calculate the probability $\delta\mathscr{P}(\alpha_f, t)$ of finding the system, in a measurement, in a given group of final states. We characterize this group of states by a domain D_f of values of the parameters α, centered at α_f, and we assume that their energies form a continuum. The postulates of quantum mechanics then yield:

$$\delta\mathscr{P}(\alpha_f, t) = \int_{\alpha \in D_f} d\alpha \, |\langle \alpha | \psi(t) \rangle|^2 \tag{C-27}$$

As in the example of §α above, we shall change variables, and introduce the density of final states. Instead of characterizing these states by the parameters α, we shall use the energy E and a set of other parameters β (which are necessary when H_0 alone does not constitute a C.S.C.O.). We can then express $d\alpha$ in terms of dE and $d\beta$:

$$d\alpha = \rho(\beta, E)\, d\beta\, dE \tag{C-28}$$

in which the density of final states $\rho(\beta, E)$* appears. If we denote by $\delta\beta_f$ and δE_f the range of values of the parameters β and E defined by D_f, we obtain:

$$\delta\mathscr{P}(\alpha_f, t) = \int_{\substack{\beta \in \delta\beta_f \\ E \in \delta E_f}} d\beta\, dE\, \rho(\beta, E)\, |\langle \beta, E | \psi(t) \rangle|^2 \tag{C-29}$$

where the notation $|\alpha\rangle$ has been replaced by $|\beta, E\rangle$ in order to point up the E- and β-dependence of the probability density $|\langle \alpha | \psi(t) \rangle|^2$.

b. FERMI'S GOLDEN RULE

In expression (C-29), $|\psi(t)\rangle$ is the normalized state vector of the system at time t. As in §A of this chapter, we shall consider a system which is initially in an eigenstate $|\varphi_i\rangle$ of H_0 [$|\varphi_i\rangle$ therefore belongs to the discrete spectrum of H_0, since the initial state of the system must, like $|\psi(t)\rangle$, be normalizable]. In (C-29), we shall replace the notation $\delta\mathscr{P}(\alpha_f, t)$ by $\delta\mathscr{P}(\varphi_i, \alpha_f, t)$ in order to remember that the system starts from the state $|\varphi_i\rangle$.

The calculations of §B and their application to the case of a sinusoidal or constant perturbation (§§C-1 and C-2) remain valid when the final state of the system belongs to the continuous spectrum of H_0. If we assume W to be constant, we can therefore use (C-6) to find the probability density $|\langle \beta, E | \psi(t) \rangle|^2$ to first order in W. We then get:

$$|\langle \beta, E | \psi(t) \rangle|^2 = \frac{1}{\hbar^2} |\langle \beta, E | W | \varphi_i \rangle|^2 F\left(t, \frac{E - E_i}{\hbar}\right) \tag{C-30}$$

* In the general case, the density of states ρ depends on both E and β. However, it often happens (*cf.* example of §α above) that ρ depends only on E.

It is equal to:

$$w(\varphi_i, \alpha_f) = \frac{2\pi}{\hbar} |\langle \beta_f, E_f = E_i \mid W \mid \varphi_i \rangle|^2 \rho(\beta_f, E_f = E_i) \tag{C-36}$$

This important result is known as *Fermi's golden rule.*

COMMENTS:

(*i*) Assume that W is a sinusoidal perturbation of the form (C-1-a) or (C-1-b), which couples a state $|\varphi_i\rangle$ to a continuum of states $|\beta_f, E_f\rangle$ with energies E_f close to $E_i + \hbar\omega$. Starting with (C-11), we can carry out the same procedure as above, which yields:

$$w(\varphi_i, \alpha_f) = \frac{\pi}{2\hbar} |\langle \beta_f, E_f = E_i + \hbar\omega \mid W \mid \varphi_i \rangle|^2 \rho(\beta_f, E_f = E_i + \hbar\omega) \tag{C-37}$$

(*ii*) Let us return to the problem of the scattering of a particle by a potential W whose matrix elements in the $\{ |\mathbf{r}\rangle \}$ representation are given by:

$$\langle \mathbf{r} \mid W \mid \mathbf{r}' \rangle = W(\mathbf{r})\, \delta(\mathbf{r} - \mathbf{r}') \tag{C-38}$$

Now assume that the initial state of the system is a well-defined momentum state:

$$|\psi(t = 0)\rangle = |\mathbf{p}_i\rangle \tag{C-39}$$

and we shall calculate the scattering probability of an incident particle of momentum $\mathbf{p}_i$ into the states of momentum $\mathbf{p}$ grouped about a given value $\mathbf{p}_f$ (with $|\mathbf{p}_f| = |\mathbf{p}_i|$). (C-36) gives the scattering probability $w(\mathbf{p}_i, \mathbf{p}_f)$ per unit time and per unit solid angle about $\mathbf{p} = \mathbf{p}_f$:

$$w(\mathbf{p}_i, \mathbf{p}_f) = \frac{2\pi}{\hbar} |\langle \mathbf{p}_f \mid W \mid \mathbf{p}_i \rangle|^2 \rho(E_f = E_i) \tag{C-40}$$

Taking into account (C-20), (C-38) and expression (C-24) for $\rho(E)$, we then get:

$$w(\mathbf{p}_i, \mathbf{p}_f) = \frac{2\pi}{\hbar} m \sqrt{2mE_i} \left(\frac{1}{2\pi\hbar}\right)^6 \left| \int d^3r\, e^{i(\mathbf{p}_i - \mathbf{p}_f)\cdot\mathbf{r}/\hbar} W(\mathbf{r}) \right|^2 \tag{C-41}$$

On the right-hand side of this relation, we recognize the Fourier transform of the potential $W(\mathbf{r})$, evaluated for the value of $\mathbf{p}$ equal to $\mathbf{p}_i - \mathbf{p}_f$.

Note that the initial state $|\mathbf{p}_i\rangle$ chosen here is not normalizable, and it cannot represent the physical state of a particle. However, although the norm of $|\mathbf{p}_i\rangle$ is infinite, the right-hand side of (C-41) maintains a finite value. Intuitively, we can therefore expect to obtain a correct physical result from this relation. If we divide the probability obtained by the probability current:

$$J_i = \left(\frac{1}{2\pi\hbar}\right)^3 \frac{\hbar k_i}{m} = \left(\frac{1}{2\pi\hbar}\right)^3 \sqrt{\frac{2E_i}{m}} \tag{C-42}$$

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so that we find

$$\sum_n P_{0\to n}(t) = \Gamma t \tag{12-19}$$

where the *transition rate* Γ is given by

$$\Gamma = \frac{2\pi}{\hbar}[|\langle n|V|0\rangle|^2 \rho(\varepsilon_n)]_{\varepsilon_n = \varepsilon_0}. \tag{12-20}$$

This formula for the transition rate was named by Fermi — as some measure of its importance — the *golden rule.*

Another way of writing the golden rule comes from noticing that as t grows, $4(\sin^2[(\varepsilon_n - \varepsilon_0)t/2\hbar]/(\varepsilon_n - \varepsilon_0)^2$ becomes more and more peaked about $\varepsilon_n = \varepsilon_0$, has total area $2\pi t/\hbar$, and therefore approaches $(2\pi t/\hbar)\delta(\varepsilon_0 - \varepsilon_n)$ [except for the small wiggles in the wings]. Thus we can write

$$P_{0\to n}(t) = \Gamma_{0\to n} t \tag{12-21}$$

where

$$\Gamma_{0\to n} = \frac{2\pi}{\hbar}|\langle n|V|0\rangle|^2 \delta(\varepsilon_n - \varepsilon_0). \tag{12-22}$$

Remember, though, that to get actual numbers from this formula we must sum $\Gamma_{0\to n}$ over a continuous group of final states

$$\Gamma = \sum_{\substack{n \\ \text{in group}}} \Gamma_{0\to n}. \tag{12-23}$$

The golden rule is not valid for all times. First of all, in order that the central bump of $P_{0\to n}$ fall within the group of final states that we are looking at, the range of energies $\Delta\varepsilon$ of these states must be larger than $2\pi\hbar/t$, i.e., we need

$$t > \frac{2\pi\hbar}{\Delta\varepsilon}. \tag{12-24a}$$

On the other hand, the time must be short enough so that many states fall within the bump, i.e., the level spacing $\delta\varepsilon$ must be small compared with $2\pi\hbar/t$, or

$$t \ll \frac{2\pi\hbar}{\delta\varepsilon}. \tag{12-24b}$$

Furthermore one must keep in mind the depletion of the initial state after long times.

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ADVANCED QUANTUM MECHANICS

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So for the transition probability per unit time into a solid angle element $d\Omega$ we obtain the famous *Golden Rule*

$$w_{d\Omega} = \int (|c_m^{(1)}|^2/t)\rho_{\hbar\omega,\, d\Omega}\, d(\hbar\omega)$$
$$= (2\pi/\hbar)\,|\langle m|\, H_I'\,|\, l\rangle|^2 \rho_{\hbar\omega,\, d\Omega}, \tag{2.117}$$

where $\hbar\omega$ must satisfy

$$E_m - E_l + \hbar\omega = 0. \tag{2.118}$$

Spontaneous emission in the dipole approximation. In the case of spontaneous emission, an atomic state A makes a radiative transition to a state B in the absence of any incident electromagnetic wave. The matrix element $\langle B\,|H_I'\,|\,A\rangle$ in this case is just (2.99) with $e^{i\omega t}$ omitted and $n_{\mathbf{k},\alpha}$ set equal to zero. Hence for $w_{d\Omega}$ we obtain

$$w_{d\Omega} = \frac{2\pi}{\hbar}\frac{e^2\hbar}{2m^2\omega V}\Big|\sum_i \langle B|\, e^{-i\mathbf{k}\cdot\mathbf{x}_i}\boldsymbol{\epsilon}^{(\alpha)}\cdot\mathbf{p}_i\,|\,A\rangle\Big|^2 \frac{V\omega^2\, d\Omega}{(2\pi)^3\hbar c^3}, \tag{2.119}$$

where ω satisfies the energy conservation $E_A = E_B + \hbar\omega$. The normalization volume V cancels out as it should.

In a typical atomic transition in the optical region the wavelength of the emitted photon is much greater than the linear dimension of the atom:

$$\lambda\!\!\!^{-}_{\text{photon}} = 1/|\mathbf{k}| \gg r_{\text{atom}}, \tag{2.120}$$

since $\lambda\!\!\!^{-}_{\text{photon}}$ is typically of the order of several thousand angstrom units whereas the atomic radius is of the order of one angstrom unit. This means that we can replace

$$e^{-i\mathbf{k}\cdot\mathbf{x}_i} = 1 - i\mathbf{k}\cdot\mathbf{x}_i - (\mathbf{k}\cdot\mathbf{x}_i)^2/2 + \cdots \tag{2.121}$$

by its leading term 1. It turns out that the spin-magnetic-moment interaction is also negligible. To see this, just note that the matrix element of $(e/mc)\boldsymbol{\epsilon}^{(\alpha)}\cdot\mathbf{p}_i$ is larger than that of $(e\hbar/2mc)\boldsymbol{\sigma}_i\cdot(\mathbf{k}\times\boldsymbol{\epsilon}^{(\alpha)})$, again by $\lambda\!\!\!^{-}_{\text{photon}}/r_{\text{atom}}$ since the matrix element of $\mathbf{p}_i$ is of the order of $\hbar/r_{\text{atom}}$. An approximation in which only the $\boldsymbol{\epsilon}^{(\alpha)}\cdot\mathbf{p}_i$ term is kept is called the electric dipole ($E1$) approximation.

To further simplify the problem let us assume that only one of the atomic electrons participates in spontaneous emission, as in the case of a hydrogen-like atom (an atom in which there is only one valence electron).‡ Omitting the sum over i we have

$$w_{d\Omega} = \frac{e^2\omega}{8\pi^3 m^2\hbar c^3}|\langle B|\,\mathbf{p}\,|\,A\rangle\cdot\boldsymbol{\epsilon}^{(\alpha)}|^2 d\Omega. \tag{2.122}$$

Meanwhile, using the commutation relation between $\mathbf{p}^2$ and $\mathbf{x}$,

$$[\mathbf{p}^2, \mathbf{x}] = -2i\hbar\mathbf{p}, \tag{2.123}$$

‡Many of the results we derive for one-electron atoms can readily be generalized to many-electron atoms.

APPENDIX D

FORMULAS AND RULES IN COVARIANT PERTURBATION THEORY

I. Definition of the $\mathscr{M}$-matrix:

$$S_{fi} = \delta_{fi} - i(2\pi)^4\delta^{(4)}(P_f^{(\mathrm{tot})} - P_i^{(\mathrm{tot})})\sqrt{\prod_{\mathrm{ext}}(n_j/V)}\,\mathscr{M}_{fi},$$

where

$$n_j = \begin{cases} m_j/E_j & \text{for fermion} \\ 1/2E_j & \text{for boson.} \end{cases}$$

II. Relation of $\mathscr{M}_{fi}$ to transition probabilities and cross sections ("covariant Golden Rule"):

a) Decay $1 \to 2 + 3 + \ldots + n$. The differential decay rate dw is

$$dw = \frac{1}{2E_1}|\mathscr{M}_{fi}|^2[\prod_{\substack{\mathrm{ext}\\ \mathrm{fermions}}}(2m_{\mathrm{fermi}})]\frac{d^3p_2}{(2\pi)^3 2E_2}\cdots\frac{d^3p_n}{(2\pi)^3 2E_n}(2\pi)^4\delta^{(4)}\Big(p_1 - \sum_{j=2}^{n} p_j\Big).$$

To eliminate the $\delta^{(4)}$ function first integrate over the (three-) momentum of one of the final-state particles, and then use

$$d^3p_k\delta\Big(E_1 - \sum_{j=2}^{n} E_j\Big) = \frac{|\mathbf{p}_k|^2 d\Omega_k}{\Big(\partial\Big(\sum_{j=2}^{n} E_j\Big)\Big/\partial|\mathbf{p}_k|\Big)_{\theta_k,\phi_k}} = \frac{|\mathbf{p}_k|^2 d|\mathbf{p}_k| d\phi_k}{\Big(\partial\Big(\sum_{j=2}^{n} E_j\Big)\Big/\partial(\cos\theta_k)\Big)_{|\mathbf{p}_k|,\phi_k}}.$$

For $1 \to 2 + 3$ the (partial) decay rate is given in the rest frame of particle 1 by

$$\Gamma(1 \to 2 + 3) = \frac{1}{2}\frac{1}{(4\pi)^2}\frac{|\mathbf{p}_{\mathrm{fin}}|}{m_1^2}\int d\Omega \sum_{\substack{\mathrm{final}\\ \mathrm{spin}}}|\mathscr{M}_{fi}|^2 \prod_{\substack{\mathrm{ext}\\ \mathrm{fermions}}}(2m_{\mathrm{fermi}}),$$

where $|\mathbf{p}_{\mathrm{fin}}| = |\mathbf{p}_2| = |\mathbf{p}_3|$.

b) Differential cross section for $1 + 2 \to 3 + 4 + \cdots + n$ when the momenta of particles 1 and 2 are collinear:

$$d\sigma = \frac{1}{v_{\mathrm{rel}}}\frac{1}{2E_1}\frac{1}{2E_2}|\mathscr{M}_{fi}|^2[\prod_{\substack{\mathrm{ext}\\ \mathrm{fermions}}}(2m_{\mathrm{fermi}})]\frac{d^3p_3}{(2\pi)^3 2E_3}\cdots\frac{d^3p_n}{(2\pi)^3 2E_n}$$

$$\times (2\pi)^4\delta^{(4)}\Big(p_1 + p_2 - \sum_{j=3}^{n} p_j\Big);$$

$$v_{\mathrm{rel}}E_1E_2 = \begin{cases} E_{\mathrm{tot}}|\mathbf{p}_{\mathrm{in}}| & \text{in the CM system} \quad (|\mathbf{p}_{\mathrm{in}}| = |\mathbf{p}_1| = |\mathbf{p}_2|) \\ m_2|\mathbf{p}_1| & \text{in the rest system of particle 2 (lab system)} \\ \sqrt{(p_1\cdot p_2)^2 - (m_1m_2)^2} & \text{in general.} \end{cases}$$

For $1 + 2 \longrightarrow 3 + 4$, the differential cross section in the CM-system is given by

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{CM}} = \frac{1}{4}\frac{1}{(4\pi)^2}\frac{1}{E_{\text{tot}}^2}\frac{|\mathbf{p}_{\text{fin}}|}{|\mathbf{p}_{\text{in}}|}|\mathscr{M}_{fi}|^2 \prod_{\substack{\text{ext}\\ \text{fermions}}} (2m_{\text{fermi}}).$$

One may sum over the final spin (polarization) states. If the initial particles are unpolarized, one may sum over the initial spin states and divide by the number of the spin states of the initial system.

III. Rules for writing $-i\mathscr{M}_{fi}$:

a) *External lines*

$\epsilon_\mu^{(\alpha)}$ for each absorbed photon (or spin-one boson)

$\epsilon_\mu^{(\alpha)}$ for each emitted photon (or spin-one boson)

(If $\epsilon_\mu^{(\alpha)}$ represents circular polarization, it should be complex-conjugated when it appears in the final state.)

1 for each spinless boson absorbed or emitted

$u^{(s)}(\mathbf{p})$ for each absorbed spin-$\frac{1}{2}$ fermion

$\bar{v}^{(s)}(\mathbf{p}) = (-1)^s\bar{u}^{(5-s)}(-\mathbf{p})$ for each absorbed spin-$\frac{1}{2}$ antifermion

$\bar{u}^{(s)}(\mathbf{p})$ for each emitted spin-$\frac{1}{2}$ fermion

$v^{(s)}(\mathbf{p}) = (-1)^s u^{(5-s)}(-\mathbf{p})$ for each emitted spin-$\frac{1}{2}$ antifermion

b) *Vertex factors*

$$-e\gamma_\mu \text{ if } \mathscr{H}_{\text{int}} = -ie\,\bar{\psi}\gamma_\mu\psi A_\mu$$
$$-iG \text{ if } \mathscr{H}_{\text{int}} = G\,\bar{\psi}\psi\phi$$
$$G\gamma_5 \text{ if } \mathscr{H}_{\text{int}} = iG\,\bar{\psi}\gamma_5\psi\phi, \text{ etc.}$$

In general, take $-i\,\mathscr{H}_{\text{int}}$ (more precisely $i\,\mathscr{L}_{\text{int}}$) and replace the field operators by the appropriate free-particle wave functions. Omit $e^{\pm ip\cdot x}$ and the factors already taken care of by external lines (for example, $\epsilon_\mu^{(\alpha)}$ and $u^{(s)}(\mathbf{p})$) and normalization constants ($\sqrt{m/EV}$). The remainder is the *vertex factor*.‡

c) *Internal lines*

Photon ("covariant photon"): $\dfrac{\delta_{\mu\nu}}{i(q^2 - i\epsilon)}$

Spin-one boson: $\dfrac{1}{i(q^2 + \mu^2 - i\epsilon)}\left(\delta_{\mu\nu} + \dfrac{q_\mu q_\nu}{\mu^2}\right)$

Spinless boson: $\dfrac{1}{i(q^2 + \mu^2 - i\epsilon)}$

Spin-$\frac{1}{2}$ fermion: $\dfrac{-i\gamma\cdot q + m}{i(q^2 + m^2 - i\epsilon)}$

‡If the interaction density involves derivatives of field operators, $\mathscr{H}_{\text{int}}$ differs from $-\mathscr{L}_{\text{int}}$. It can be shown (using an argument originally given by P. T. Matthews for the case of the pseudovector coupling of a pseudoscalar field) that the vertex factor in the Feynman diagram should be read directly from $i\,\mathscr{L}_{\text{int}}$ rather than from $-i\,\mathscr{H}_{\text{int}}$. In this connection we may remark that it is possible to formulate an S matrix expansion using $\mathscr{L}_{\text{int}}$ (without recourse to $\mathscr{H}_{\text{int}}$). See Bogoliubov and Shirkov (1959), pp. 206–226.

Quantum Mechanics

Volume I: Fundamentals

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such arguments is that the transition rate between energy eigenstates is

$$\text{“}\dot{P}_{ba}\text{”} = \frac{2\pi}{\hbar} |T_{ba}|^2 \, \delta(E_b - E_a). \tag{33}$$

In the lowest approximation to T, i.e., $T \simeq H'$, (33) reads

$$\text{“}\dot{P}_{ba}\text{”} = \frac{2\pi}{\hbar} |H'_{ba}|^2 \, \delta(E_b - E_a). \tag{34}$$

In order to get a sensible (i.e., finite) number from (34), one sums over final states having an energy in the immediate vicinity of E_b, and thereby defines a total transition rate to this entire group of states by

$$\dot{P}_{Ba} = \frac{2\pi}{\hbar} |H'_{ba}|^2 \rho_B. \tag{35}$$

This formula of Dirac has given such faithful and meritorious service that Fermi called it the Golden Rule. We shall also use this terminology.

The cross section is determined from (35) by dividing by the incident flux. As the initial state is now a plane wave state normalized to one in a box of volume Ω, the flux is given by v_a/Ω. The cross section is therefore

$$d\sigma_{Ba} = \frac{2\pi}{\hbar} \frac{\Omega}{v_a} |H'_{ba}|^2 \rho_B. \tag{36}$$

This agrees with (31) when the Born approximation $T \simeq H'$ is made in the latter expression.

57. *Collision Phenomena in the Born Approximation*

We shall now apply the Golden Rule to a variety of collision phenomena. Our aim here is to illustrate the power and limitations of the technique, and to establish its relationship to the treatment of elastic scattering given in Chapt. III.

At the outset we shall study a system that does not really occur in nature, but which does constitute a prototype for more complex processes that abound in atomic and nuclear physics. The model consists of two spinless and distinguishable particles designated by α and β which interact via a central potential $V(|\mathbf{r}_\alpha - \mathbf{r}_\beta|)$. Beyond this force there is also a fixed, attractive, central field that only acts on β. The full Hamil-

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Tung-Mow Yan

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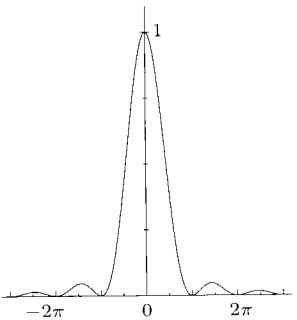

FIG. 3.5. The function $f(x) = \sin^2 x/x^2$ that occurs in time-dependent perturbation theory. Note that $T^2 f(\omega T)$ appears in (310) and (312).

or

$$\lim_{T\to\infty} \frac{\sin^2 \omega T}{\omega^2} = \pi T\, \delta(\omega)\ . \tag{312}$$

The transition probability therefore tends to the following result as $T \to \infty$:

$$P_{i\to f} = \frac{2\pi}{\hbar}\, |\langle \Phi_f | V | \Phi_i \rangle|^2\, \delta(E_f - E_i) \cdot (2T)\ . \tag{313}$$

Thus this probability is proportional to the time interval $2T$ over which the interaction is active in virtue of the way it is idealized by (309). Equation (313) is therefore the consequence of the steady *transition rate*

$$\dot{P}_{i\to f} = \frac{2\pi}{\hbar}\, |\langle \Phi_f | V | \Phi_i \rangle|^2\, \delta(E_f - E_i)\ . \tag{314}$$

This is Dirac's result; it is the most important result of time-dependent perturbation theory. As we shall learn in §9.1(c), its form holds to all orders in V. Fermi called (314) the *Golden Rule*, and it is often wrongly attributed to him. Note that (314) has the dimension (Energy/$\hbar$) if the states are normalized to unity, i.e., the dimension of 1/time, as required for a rate.

The transition rate is, among other things, an expression of *the law of energy conservation*, as it should be because it describes processes (in our example elastic scattering) due to time-independent interactions. When the interaction time $2T$ is finite, energy conservation is not perfect; indeed, as one sees from Fig. 3.5, the spread ΔE of values of $|E_f - E_i|$ is approximately

$$T\, \Delta E \gtrsim C\hbar\ , \tag{315}$$

where C is of order unity. This is an example of the time-energy uncertainty relation. In contrast to the uncertainty relations for incompatible observables, it does not have an unambiguous lower limit, something we already learned in §2.4(c).

Equation (314) gives the transition rate between states of perfectly sharp energies. In any actual experiment, the resolution is always finite, and the result observed is a sum over states in some interval $(E_f, E_f + dE_f)$. Let dN_f be the number of states in this interval. This number depends on the nature of the final states $|\Phi_f\rangle$ — whether they contain one or more particles. Here we take the simplest case, a

Nuclear Physics

A Course Given by ENRICO FERMI
at the University of Chicago. Notes Compiled by
Jay Orear, A. H. Rosenfeld, and R. A. Schluter

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PREFACE

This material is a reproduction, with some amplification, of our notes on lectures in Physics 262-3: Nuclear Physics, given by Enrico Fermi, Jan.-June 1949. The course covered a large number of topics, both experimental and theoretical.

The lectures presupposed a familiarity with physics generally acquired by a student who has completed one course in quantum mechanics (this to include a discussion of the Pauli spin operators and of perturbation theory, both time-independent and time-dependent). We shall make some use of elementary concepts of such topics as statistical mechanics and electrodynamics, but we give references, and a reader could probably pick up the necessary ideas as he goes along; or he could omit a few sections.

Dr. Fermi has not read this material; he is not responsible for errors. We have made some attempt to confine the classroom presentation to the text proper, putting much of our amplifications in footnotes, appendices, and in the solutions to the problems. Most of the problems were assigned in class, but the solutions are not due to Dr. Fermi.

The literature references in the text apply to the list on page 239. At the end of the book there is also a summary of the notation and a list of pertinent constants, values, and relationships.

We would very much appreciate your calling errors to our attention; we would like to hear any suggestions and comments that you may have.

May we thank warmly all those who have helped us to prepare these notes.

Jay Orear
A.H. Rosenfeld
R.A. Schluter

January, 1950

This second printing of these notes differs from the first in that corrections and minor revisions have been made on approximately 70 pages in the first nine chapters, and major revisions have been made in the chapter on cosmic rays. We are grateful to the many people who have given suggestions and corrections; in particular, we are indebted to Prof. Marcel Schein for his suggestions and generous aid in revision of Chapter X.

JO, AHR, RAS

September 1950

An attempt to bring this second printing of the revised edition up to date has been made by adding new footnotes and two pages (237,238) of recent developments. Corrections and minor revisions have been made on approximately 40 pages.

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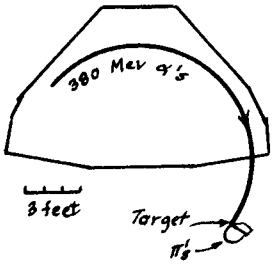
CHAPTER VII. MESONS

A. PROPERTIES KNOWN FROM EXPERIMENT

In this section we shall discuss briefly some of the facts known about mesons, and summarize them in a table; except for one of the problems, however, we shall not discuss the experiments behind the facts*.

By mesons we mean unstable particles of mass greater than that of the electron, less than that of the nucleon. The only ones directly observable so far have either a positive or negative fundamental charge.

Mesons were postulated by Yukawa in 1935, and soon thereafter μ-mesons (they will be called "μ's" or muons from here on) were observed as secondary particles in cosmic radiation***. In 1948 π-mesons (π's or pions) were created artificially by bombarding various targets in the Berkeley cyclotron**. During 1949-50, overwhelming evidence has been found for the existence of a neutral pion π^0. This is discussed further on p. 237



Production of $\pi^{\pm}$'s in a cyclotron.

So far only two sorts of mesons, π and μ, have been identified beyond all doubt, but there are rumors of others.

The names ρ and σ are also used in the literature. This is because the various kinds of meson tracks observed were classified phenomenologically by Powell and his associates according to what was observed at the end of the tracks. This nomenclature is confusing because the number of different kinds of mesons turned out to be less than the number of categories chosen, so that identical mesons may be called by different names.

A ρ meson is one which is observed to stop in the emulsion without producing any observable product. This is a rather time-dependent definition, since more sensitive films are currently being developed. Thus previously unobservable singly-charged relativistic particles (particles travelling at "minimum ionization" -- see Fig. II.4, p. 33) may now be detected.

A σ meson (σ for "star-producing") denotes a meson which produces a nuclear disintegration at the end of its track.

*For nice discussions see "Mesons Old and New" by Keller, Am. Jour. Phys. 17, 356 (Sept. 1949) and a 10-page article by Snyder, Nucleonics 5, 42 (July '49). See also Occhialini and Powell, "Nuclear Physics in Photographs" (1947); and all the references on p. 239 of this book.

**Gardner and Lattes, Phys. Rev. 74, 1236 ('48), Science 107, 270 ('48); Burfening and Lattes, Phys. Rev. 75, 382 ('49).

***Neddermeyer and Anderson, Phys. Rev. 51, 884 ('37), Street and Stevenson, Phys. Rev. 51, 1005 ('37).

The π-Meson (π for "primary" -- for a summary of its properties, see TABLE VII.I):

1. Charged Pions.

The mean life, $\tau \sim 10^{-8}$ sec*, given in the table, applies in the c-m system of the π. Observed in the laboratory system, this time appears dilated by a factor of $\gamma = (1-\beta^2)^{-1/2} = W/M_\pi c^2$. Therefore a $\pi^\pm$ formed with an energy of several Bev during a collision of a high energy cosmic ray particle and a nucleus could travel, at a speed approaching that of light, many meters before it decays. In this case it will probably decay at high energy (before it slows down), into a high-energy μ and a neutrino (?).

If a $\pi^\pm$ slows down before decaying (or is formed at low energy) then as it slows down to about 10 Mev, its rate of ionization increases slowly to about five times minimum ionization, at which point it becomes visible even in the older nuclear films. The last 10 Mev of its path is about 2500 microns long.

A π^+, which is repelled by nuclei, simply comes to rest and decays. But a slow π^- is attracted****and frequently absorbed by a nucleus, giving up its rest energy and probably boiling off several nucleons. These two sorts of tracks are illustrated in FIG. VII.1.

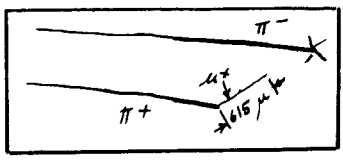


FIG. VII.1 Tracks of π Mesons in Emulsion.

When not captured by a nucleon, a π decays as follows:

$$\pi^\pm \xrightarrow{10^{-8}\ \text{sec}} \mu^\pm + \nu'$$

where ν' is thought to be a neutrino (we shall refer to it as such). As illustrated in problem 1, p. 138, $M_{\nu'}c^2$ is known to be < 15 Mev.

2. Neutral Pions -- see p. 237.

The μ-Meson (again, see TABLE VII.I for mass, etc.):

If the decay reaction mentioned just above takes place while the π is at rest, the μ has a kinetic energy of 4.1 Mev and travels almost exactly 615 microns in Ilford emulsions. Of course, most of the μ's in cosmic radiation are formed when π's decay at high energy, so their range is >> 615 microns.**

On most film the end of the path looks blank, but with cloud chambers, g-m tubes, or minimum ionization film, it has been determined that, when there are no heavy nuclei around, one of the products of the μ-disintegration is an electron which may have one of several energies and is thought to have a continuous spectrum from 9 to 55 Mev***. No other particles have been detected during the reaction, so that the most logical guess is

$$\mu^\pm \xrightarrow{2.15\ \mu\text{sec}} e^\pm + 2\nu \quad (\text{i.e. at least } 2\nu) \qquad \text{in vacuo}.$$

The electron is so light compared to the μ that, on the average, we can think of the energy as being essentially divided equally among the three particles, all extremely relativistic.

*Richardson, Phys. Rev. 74, 1720 ('48)

**An energy spectrum of cosmic ray μ's is given in FIG. X.5, p.220.

***Steinberger, Phys. Rev. 75, 1136 ('49) and Leighton, Anderson, Seriff, Phys. Rev. 75, 1432 ('49): Current data is inadequate for differentiation between several discreet energies (as for α's) and a continuum.

****See the discussion at the top of p. 133.

The qualification "in vacuo" concerning the mean life of u's is needed because, in matter, a positive meson is repelled by nuclei, but a negative meson may fall into a stable Bohr orbit just as an electron does***. The orbits have radius and energy

$$r = \frac{(n\hbar)^2}{M Z e^2} \qquad E = -\frac{M(Ze^2)^2}{2(n\hbar)^2}$$

where M is the reduced mass. The μ orbit is smaller than the corresponding electronic orbit by a factor of 216, and the binding energy is increased by the same factor. For heavy elements, the smallest orbit is only slightly larger than the nucleus itself, so that the μ spends a large fraction of its time inside the nucleus. If the μ interacted strongly with the nucleons (as a π does) it would be immediately captured by the nucleus, but we find that the interaction is very weak. Ticho * gives a curve showing that τ_μ-drops from 2.15 μsec for Z = 1 to 0.7 μsec for Z = 16 , where the capture probability has started to compete seriously with the natural decay. Remember that these mean lives apply to the c-m system of the meson.

TABLE VII.I summarizes the material discussed in this section:

	Electron Masses	Mc^2	Probable Spin	MEAN Life in Vacuo	Interaction with Nuclei
$\pi^\pm$	276 ± 6	136 Mev	0 or 1	$\sim 10^{-8}$ sec	Strong, → exchange forces
$\mu^\pm$	210 ± 4	107 Mev	$\frac{1}{2}$	2.15 usec	Weak, → exchange forces
π^o	≈276 - 6 see p.237	135 Mev	0	$<10^{-13}$ sec	Strong, → ordinary forces

	Path Length in Emulsion	Decay Products
$\pi^\pm$	Non-relativistic ~2500μ. (see text)	π^- usually → star in film $\pi^+ \longrightarrow \mu^+$ (4.1 Mev) + ν
$\mu^\pm$	615 μ (μ ≡ micron)	$e^\pm$ (<55 Mev) + 2ν (?)
π^o	not observable	2 photons

TABLE VII.I Mesons.**

B. MESON THEORY

From electrostatics we know that two particles attract or repel one another according to Coulomb's law. For a classical treatment we say that this force arises from the potential field $\phi = e/r$ of one of the particles. However if we wish to take into account the corpuscular nature of light, we can describe this interaction by saying that one particle"emits" a photon which is subsequently absorbed by the other.

Analogously, the interaction of two nucleons can be partially interpreted by the picture of one nucleon "emitting" a quantum which is promptly absorbed by the second nucleon. These quanta are called mesons, and we shall call them π-mesons in this discussion. The reason for this nomenclature is that we know experimentally that nucleons interact

*Phys. Rev. 74, 1337 ('48)
**This footnote has been expanded and put on p. 237
***Fermi and Teller, Phys. Rev. 72, 399 ('47), J.A. Wheeler, Rev. Mod.Phys. 21, 133 ('49)

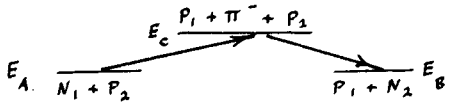
$$e^{-\kappa r}\left[\frac{1}{r}, \frac{1}{r^2}, \frac{1}{r^3}, \cdots\right]$$

with some directional terms.

Unfortunately it is impossible to find a solution for Schrödinger's equation when the potential diverges faster than $1/r^2$ at the origin. Where $1/r^3$ terms appear, the field must be arbitrarily cut off in a finite volume, but this makes it impossible to formulate the problem in a relativistically invariant way.

Because of these difficulties there are as yet no self-consistent results from meson theory.

In order to point out another important difficulty in meson theory, we must now discuss, exceedingly briefly, the quantum-mechanical formulation of the problem. As an example we shall take one of the "reactions" postulated on p. 134. We have illus-



$$\left.\begin{array}{ll} N_1 \rightarrow P_1 + \pi^- & (A \rightarrow C) \\ \pi^- + P_2 \rightarrow N_2 & (C \rightarrow B) \end{array}\right\}$$

trated in the sketch at left that the intermediate state (C) is energetically impossible for nucleons at rest, since it "costs" 145 Mev to create a π^-. In quantum mechanical perturbation theory, however, states with energies above or below that of the system are important as intermediate, or virtual, states. We shall make extensive use of intermediate states (for example in Ch. VIII to derive the Breit-Wigner formula). Since the mean life of the intermediate state is short ($\tau \sim \hbar/\Delta E$ by the uncertainty principle) there is no violation of conservation of energy.

The transition probability and energy perturbation can be calculated with the help of perturbation theory (ie., there is no better way known). Since the direct matrix element coupling the initial and final states is assumed to be zero, we use "Golden Rule #1" for the second order transition:

$$H'_{BA} = \sum_C \frac{H'_{BC}\, H'_{CA}}{E_A - E_C}$$

Now we can point out the difficulty. It turns out that only the first non-vanishing matrix element (in this example the second-order one) is finite, but that the higher order elements are sums that are not negligible- in fact they diverge. The divergences in the corresponding terms in the electromagnetic case can be removed relativistically by the recent advances in quantum mechanics, but the way out of the difficulty has not been found in meson theory.

Even if the divergences of the individual higher-order transitions could be removed there is another difficulty. Perturbation theory applied to the electromagnetic case gives an expansion of successive orders of the interaction Hamiltonian in powers of $(e^2/\hbar c) = 1/137$. This parameter is quite small, so that there is hope that the whole series will converge. But meson perturbation theory is an expansion in powers of $(g^2/\hbar c)$. This cannot be made smaller than about 1/5 if the theory is to give the right magnitude of nuclear forces. There is considerably less hope that the entire series will converge, even if the individual terms can be made finite.

Meson Theory and Beta Decay. By writing in sequence the reactions

$$N \rightarrow P + (\text{Meson})^-; \qquad (\text{Meson})^- \rightarrow e^- + n\nu \quad [n = 1, 2, 3 \ ?]$$

or $P \rightarrow N + (\text{Meson})^+$, etc.

Yukawa hoped to explain β-decay. Now that it is known that there are two sorts of mesons (maybe more), only one of which decays into an electron

$$\mu^\pm \rightarrow e^\pm + 2\nu$$

it is difficult to reconcile β-decay with the known meson mean lives in a quantitative way.

Summary A great deal of attention has been given to meson theories, from which has come relatively little quantitative results. Qualitatively, however the theory is valuable. Thus physicists predicted the creation of mesons during high-energy collisions before mesons had ever been observed. Meson theory was of considerable weight in the decision to build the large synchro-cyclotrons. Another example of the qualitative application of meson theory is the discussion in Ch. I (p. 14) where we obtain a numerically wrong but qualitatively useful value for the magnetic moment of the deuteron by assuming that part of the time

$$P \rightarrow N + \pi^+$$
$$N \rightarrow P + \pi^-$$

The formalism of meson theory may be greatly modified or abandoned, but the fundamental ideas are likely to survive.

REFERENCES for further reading on meson theory.

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Yukawa, H., Proc. Math. Phys. Soc. Japan 17, 48 ('35)
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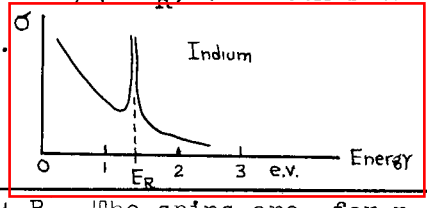
Problem: Design an experiment to detect the inverse reaction to $Be^9 + H^1 \longrightarrow Li^6 + He^4$.

(Design of the alpha particle source will depend on the threshold energy for the inverse reaction. From Allison, Skaggs and Smith, Phys.Rev. 57 550, or from Hornyak and Lauritsen, Rev. Mod.Phys. 20, 202, we find that Q for the forward reaction is 2.115 Mev. In the reverse reaction, in order to get 2.115 Mev into the center of mass coordinate system we must give the alpha an energy of about 3.5 Mev, and this is the threshold for the inverse reaction (See section A). Design of the Li^6 target and of the detector, and determining the required alpha beam strength require knowing the cross section. This is got by detail balancing arguments from $\sigma_{Be^9(p,\alpha)Li^6}$, taking into account a spin factor of 8/3. This cross section is found in Livingston and Bethe, C, Rev.Mod.Phys. 9 245, p. 310, or in the original source, Allen, Phys.Rev. 51 182 (1937), and is 5 x 10^{-29} cm^2 at 0.1 Mev. The cross section for the inverse reaction increases rapidly as the volume of phase space available to the proton is increased, therefore it is advantageous to use alpha energies an Mev or more above the threshold of 3.5 Mev. Higher energy protons also penetrate the Coulomb barrier readily, and are easier to detect. A qualitative curve of cross-section for the forward reaction as a function of energy is given in Hornyak and Lauritsen, Rev. Mod.Phys. 20 191, p. 201.

D. The Compound Nucleus

In the diagrams of section B it was assumed the $|\mathcal{H}|^2$ was approximately constant, except for the Coulomb barrier factor. Often, perhaps in most cases, the matrix element has irregular variations. This phenomenon is called resonance. For example, in the (n, γ) process in indium, there is an extremely pronounced peak in σ at a neutron energy of 1.44 e.v. σ reaches 27,000 barns at this energy. (one barn is 10^{-24} cm^2.) The half-width of this resonance peak is 0.042 e.v. $\equiv \Gamma_{1/2}$ Near the resonance, the curve of σ vs. energy has the form $1/(E-E_R)^2$. Another example is the resonance at E_R = 5.2 e.v. for the (n, γ) reaction in silver. In this case σ reaches 24,000 barns, and the peak has a half-width $\Gamma_{1/2}$ = 0.063 e.v.



consider the reaction n + A $\longrightarrow \alpha$ + B. The spins are, for n, 1/2; for α, 0; assume for A, 1; and for B, 3/2. The total number of initial spin states = (2(1)+1)(2(1/2)+1) = 6. The number of initial spin states for total angular momentum S = 3/2 is (2(3/2)+1) = 4; for S = 1/2, (2(1/2)+1) = 2.

$$\sigma_{A(n,\alpha)B} = \frac{4}{6} \times \sigma_{(S=3/2)} + \frac{2}{6} \times \sigma_{(S=1/2)}$$

Now the first term represents transitions to any of the final spin states having S = 3/2. For a given initial orientation, there is only one. Similarly, the second term represents transitions to any final state having total angular momentum 1/2. But, since the spin of the α = 0, there are none, so $\sigma(1/2) = 0$. When orbital angular momentum is involved, there may be more than one way in which the given initial state can form a final state, so that $\sigma(3/2)$, for example, is a sum over the various possibilities. See Bethe and Placzek, Phys. Rev. 51 450, appendix.

The explanation of this phenomenon is based on the assumption that the transition A + a → B + b occurs through an intermediate state C:

$$A + a \longrightarrow C \longrightarrow B + b$$

State C is the "Compound nucleus". The idea of the compound nucleus is due to Bohr.*

The idea of how resonances in cross section result from this assumption can be obtained from the quantum mechanics of second order transitions. The probability of transition, per unit time, is given by "Golden Rule No. 1":**

$$\text{trans. prob./sec} = \frac{2\pi}{\hbar}\left|\frac{\mathcal{H}_{cA}\mathcal{H}_{Bc}}{E_A - E_c}\right|^2 \times \begin{pmatrix}\text{energy}\\ \text{density of}\\ \text{states}\end{pmatrix} \qquad \text{VIII.19}$$

provided there are no direct transitions from A to B. The cross section is, from VIII.9,

$$\sigma_{A\to B} = \frac{1}{\pi\hbar^4}\left|\Omega\mathcal{H}\right|^2 \frac{p_b^2}{v_a v_b} \qquad \text{VIII.9'}$$

which becomes, analogously,

$$\sigma_{A\to B} = \frac{1}{\pi\hbar^4}\left|\frac{\mathcal{H}_{cA}\mathcal{H}_{Bc}}{E_A - E_c}\right|^2 \frac{p_b^2}{v_a v_b} \qquad \text{VIII.20}$$

Near $E_A = E_C$, (resonance), σ is large. This formula gives infinite σ at the resonance energy, but the formula does not take into account the short lifetime of the compound state. A correct formula is derived in section F.

The life-time of the compound state is long enough for the nucleus C to "forget" how it was formed,*** and this results in a basic simplification in the interpretation.

From the Heisenberg relation $\Delta t \Delta E \gtrsim \hbar$, the lifetime of the compound nucleus and the uncertainty Γ in its energy are related by

$$\Gamma \gtrsim \frac{\hbar}{\text{lifetime}} \qquad \text{VIII.21}$$

The reasons why the compound nucleus has a lifetime greater than zero are the following:

1) For charged particle decay, the barrier factor (VIII.12) reduces the rate of decay.

2) Decay by γ radiation is very slow compared to the times in which the nucleus changes its organization: the lifetime against γ emission is $\sim 10^{-13} - 10^{-14}$ sec. The characteristic time of the nucleus, i.e., the time for a nucleon to cross the nucleus, is ~(size)/(velocity) $\approx 10^{-13}/10^{9}$, or about 10^{-22} sec.

3) A particularly important reason is the tendency toward equipartition of energy in the nucleus. The excess energy due to the absorption of the bombarding particle is distributed among all the nucleons. It is rare that there is a fluctuation in which a large fraction of the excess energy is on one nucleon.

4) Selection rules forbid some modes of decay.

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* Bohr, Nature 137 344 (1936)

** Schiff, p. 196, eq. (29.20)

*** Discussed in Peierl's review article in Reports on the Progress in Physics VIII (1940), Phys. Soc. of London, 1941.

E. Example of an Unstable Nucleus

An example of a nucleus which plays the role of an intermediate-state compound-nucleus for several well known nuclear reactions is Be^8.

The ground state Be^8 decays as follows:

$$Be^8 \longrightarrow 2\ He^4 + 110\ Kev.*$$

The reaction is barely exothermic. The Gamow exponent for decay into α's is low due to low nuclear charge, see equation VIII.57, p.163. The theoretical estimate of the lifetime is 10^{-16} sec.**, corresponding to a width of between 1 and 100 e.v. This time is long compared with the nuclear characteristic time of 10^{-22} sec.; hence the width of the level is small.

Information on the excited levels of Be^8 can be obtained from study of those nuclear reactions for which Be^8 is the intermediate compound nucleus state, such as $Li^7(p,\gamma)Be^8 \longrightarrow 2\alpha$, $Li^7(p,n)Be^7$. These reactions are discussed here. The energy levels are plotted in FIG. VIII.1.

1) α-α scattering. For two Coulomb centers, the total scattering cross section is ∞. We may study the scattering at some angle not near 0 (90° in center of mass system is best). We expect peaks in the value of σ when the incident relative energy equals the energy of excitation of an excited state. For α's scattered on α's, the first such resonance should come at 0.110 Mev (in center of mass system), corresponding to the Be^8 ground state. This resonance is presumably very sharp, a few e.v. wide, as mentioned above. It has never been observed experimentally.

Problem. Discuss the possibility of experimentally observing the resonance expected in alpha-helium scattering at an energy corresponding to the Be^8 ground state, i.e., 0.110 Mev in the center of mass frame.
(The Coulomb barrier keeps alphas of this energy at least 5 x 10^{-12} cm apart classically, so the effect of nuclear forces is probably undetectable. Also the experiment is difficult because the range of 200 Kev alphas is so short that it is hard to shoot them through an appreciable number of scattering centers and detect them. Any attempt to detect a resonance might be guided by the experimental procedure of Devons (Proc.Roy.Soc. A 172 127 and 559 (1939)), who investigated alpha-helium scattering at higher energies. The theory of α-α scattering and its relation to the Be^8 nucleus is given in Wheeler, Phys.Rev. 59 16 and 27, (1941).)

A second resonance, this one experimentally observed, is at ~3 Mev. The barrier factor is lower at 3 Mev, hence the state has shorter lifetime and greater width. The half-width is estimated to be 0.8 Mev.

Further resonances in α-α scattering are so broad as to be scarcely recognizable as resonances. All the resonances mentioned so far correspond to states of even parity. This is because α's obey Bose-Einstein statistics and have symmetric wave

* Hemmendinger; quoted in Seaborg and Perlman table of isotopes, Rev. Mod. Phys. 20 585.
** Wheeler, Phys.Rev. 59 27.

CHAPTER VIII NUCLEAR REACTIONS

A. Notation

The nuclear reaction $A + a \longrightarrow B + p + Q$ is symbolized by

$$A(a,p)B$$

Particles are symbolized by: α alpha, p proton, d deuteron, γ gamma ray, and f for fission.

Q is (+) for an "exothermic" reaction, (-) for "endothermic".

The threshold is the minimum energy of the bombarding particle in order for the reaction to occur. Threshold is measured in the laboratory system, and therefore is not necessarily equal in magnitude to Q. If Q is positive, the threshold is, in principle, 0. If Q is negative, and if the bombarded particle A is approximately at rest, then (see Ch. I, page 5)

$$\text{Threshold energy} = (-Q) \times \frac{\text{Mass of incident particle}}{\text{Reduced mass of system}}$$

$$= (-Q) \times \frac{M_a + M_A}{M_A} \qquad \text{VIII.1}$$

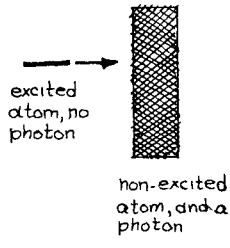
for the reaction symbolized above.

B. General Features of Cross-sections for Nuclear Reactions.

The following considerations apply to cross-sections for nuclear reactions in the absence of resonances. Resonance phenomena are discussed in section D.

Consider the transition $A + a \longrightarrow B + b + Q$, where the nucleus "A" and the particle "a" become the nucleus "B" and particle "b" Both the initial and final states of the system consist of a pair of unbound particles; therefore the transitions is to one of a continuous distribution of states. The initial state also has a continuous range of possible energies, but the experiment itself specifies a particular initial energy.

There are similar situations in atomic physics. For example, in emission of a photon by an excited atom, the transition is from a single state to one of a continuum of states: Conservation of energy selects the final state.



Another atomic example is the non-radiative or Auger transition. An excited atom may have two possible modes of decay. In addition to photon emission, the atom may decay by emission of an electron. Suppose, for example, the excitation corresponds to one missing electron in the K shell. The energy made available when an electron falls into this hole may be greater than the ionization energy, in which case an electron may be emitted from the atom. Again the final system consists of two unbound particles having a continuous range of possible energies.

Returning to the nuclear reaction $A + a \longrightarrow B + b$, we use a general principle of quantum mechanics to derive some essentially statistical results on the variation of the cross-section.

From quantum mechanics, the probability per unit time of

transition = number of transitions per unit time = w is given by "Golden Rule No. 2": *

$$w = \frac{2\pi}{\hbar} |\mathcal{H}|^2 \frac{dn}{dE} \qquad \text{VIII.2}$$

where $\mathcal{H}$ is the matrix element of the perturbation causing the transition, and dn/dE = energy density of final states, counting each degenerate state separately.

$|\mathcal{H}|^2$ may be the same for all energetically possible final states; more often it depends on the state. (For instance, $|\mathcal{H}|^2$ may depend on the direction of emission.) Then $|\mathcal{H}|^2$ in the formula is a suitable average over the possible final states.****

$dn/dE = \infty$ for a continuum of states. But in that case $|\mathcal{H}| \longrightarrow 0$, so that the expression $|\mathcal{H}|^2\, dn/dE$ has the indeterminate form $0 \times \infty$. This difficulty is removed by limiting space to a box of volume Ω. $|\mathcal{H}|$ is then small but finite and dn/dE large but finite. Ω drops out of the result. The number of final states equals the number of states of the emitted particle. This is because a change in momentum of one particle compels a change in momentum of the other, by conservation of linear and angular momentum of the system.

It was shown in Chapter IV, p. 76 that the number of states available to a free particle, "b", with momentum between p and p + dp, confined to a box of volume Ω, is

$$dn = \frac{4\pi p_b^2\, dp_b\, \Omega}{(2\pi\hbar)^3} \qquad \text{VIII.3}$$

This must be multiplied by the multiplicity in the final state** caused by spin orientation, which is given by the factor $(2I_b+1)\times(2I_B+1)$, where I_b is the spin of the emitted particle and I_B the spin of the nucleus. If b is a photon, $(2I_b+1)$ is put equal to two.***

$$dE = v_b\, dp_b \quad \text{(true relativistically)} \qquad \text{VIII.4}$$

where p_b and v_b are the momentum and velocity in the center of mass frame of reference of the final (B+b) state. Since "B" is usually massive compared with "b", p_b and v_b can usually be measured in the laboratory frame. Combining these two equations:

$$\frac{dn}{dE} = \frac{4\pi p_b^2\, \Omega}{(2\pi\hbar)^3 v_b} (2I_b+1)(2I_B+1) \qquad \text{VIII.5}$$

From this and VIII.2 we get

$$\text{No. transitions per unit time} = \frac{1}{\pi\hbar^4} \frac{p_b^2}{v_b} \Omega |\mathcal{H}|^2 (2I_b+1)(2I_B+1) \qquad \text{VIII.6}$$

The following equation is essentially a definition of the cross-section $\sigma_{A\to B}$ per A nucleus:

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* Derived in Schiff, Quantum Mechanics, p. 193. ("Golden Rule No. 1" is on page 148 of this text).

** This is discussed in greater detail in section C, this chapter.

*** This point is discussed by Bethe and Placzek, Phys.Rev. 51 450, Appendix, p. 483. Multiplicity is caused by the two possible independent polarizations.

**** See page 214 for more complete discussion.

$$\text{No. transitions/sec per "A" nucleus} = n_a \times v_{\text{"a" rel. to "A"}} \times \sigma_{A\to B} \qquad \text{VIII.7}$$

where A and B refer to the (A+a) and (B+b) states respectively, and n_a is the density of particles "a". Take n_a to be $1/\Omega$ cm^{-3} (one particle in the volume). Then

$$\frac{1}{\Omega} \times v_{\text{"a" rel. to "A"}} \times \sigma_{A\to B} = \frac{1}{\pi\hbar^4} \frac{p_b^2}{v_b} \Omega |\mathcal{H}|^2 (2I_b+1)(2I_B+1) \qquad \text{VIII.8}$$

Since nucleus "A" is often massive compared to "a", $v_{\text{"a" rel. to "A"}}$ is often nearly equal to v_a in the center of mass frame. In any case, these two velocity magnitudes are related by a constant factor. Writing $v_{\text{"a" rel. to "A"}} = v_a$,

$$\boxed{\sigma_{A\to B} = \frac{1}{\pi\hbar^4} |\Omega\mathcal{H}|^2 \frac{p_b^2}{v_a v_b} (2I_b+1)(2I_B+1)} \qquad \text{VIII.9}$$

In general, $\mathcal{H}$ is unknown. It has the form $\int d\tau \psi^*_{final} U \psi_{initial}$ where U is the interaction energy. If the wave functions used to compute $\mathcal{H}$ are normalized in volume Ω, Ω disappears from the expression $|\Omega\mathcal{H}|$ in VIII.9. This is seen as follows: Let Ψ have the form, at large distances, N exp(ikz). Then $\int |\psi|^2 d\tau = N^2\Omega$ Setting $N^2\Omega = 1$, we get $N = 1/\sqrt{\Omega}$

If $\Psi_{initial}$ and Ψ_{final} now mean the un-normalized plane wave functions, the matrix element factor in VIII.9 becomes

$$\Omega\mathcal{H} = \int d\tau \Psi^*_{final} U \Psi_{inital} \qquad \text{VIII.10}$$

(This may be looked upon as taking $\Omega = 1$.) Henceforth we use $\mathcal{H}$ for $\Omega\mathcal{H}$. In order to show the meaning of this expression, we write it as

$$|\mathcal{H}| = \bar{U} \times \text{Volume of nucleus} \times \overline{|\Psi_{initial}\Psi_{final}|} \qquad \text{VIII.11}$$

where $\overline{|\Psi_{in.}\Psi_{fin.}|}$ is a suitable average of the product of the wave functions over the volume of the nucleus. U, and hence the integrand, is zero outside the nucleus. $\bar{U}$ = average interaction energy $\approx$ depth of potential well. For our purposes here the important feature of VIII.11 is its dependence on the charge of the participating particles. If "a", say, is positively charged, its wave function will be reduced in <u>amplitude</u> at the nucleus by the barrier factor exp($-G_a/2$), where, by III.3, p. 58.

$$\frac{G_a}{2} = \sqrt{\frac{2M_a}{\hbar^2}} \int \sqrt{U_a - E_a}\, dr \longrightarrow \approx \frac{\pi Z_A g_a e^2}{\hbar v_a} \quad \text{for } \underline{\text{high}} \text{ barriers} \qquad \text{VIII.12}$$

U_a denotes the charge of "a" times the Coulomb potential of "A". Physically this factor represents Coulomb repulsion. The wave function of an outgoing particle at the nucleus is also reduced by such a barrier factor. The result for the <u>squared</u> matrix element is:

For neutral particles: $|\mathcal{H}|^2 \propto (\bar{U} \times \text{Vol. of nucleus})^2$ VIII.13
For + charged particles: $|\mathcal{H}|^2 \propto (\bar{U} \times \text{Vol.})^2 \times \exp(-G_a - G_b)$

Section 5.12: Fermi's Golden Rule

(to be replaced with more general form not referring to photons)

Fermi's Golden rule gives the *rate* of transition from a single state to a set of states, which can be described by the "density of state" function. The first topic in this section introduces the intuitive meaning and Chapter 7 continues with more detail.

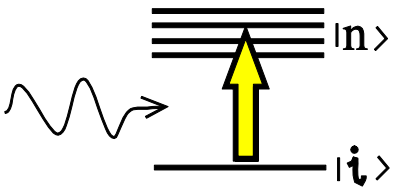

Figure 5.12.1: Schematic illustation of an electromagnetically induced transition from an initial state i to one of the final states n.

As shown in the figure, an electron makes a transition from an initial state $|i\rangle$ to one of the many final states $|n\rangle$. The probability of transition must be given by

$$\text{Total Prob} = \sum_n P(i \to n) \tag{5.12.1}$$

For a semiconductor, the final states closely approximate a continuum. In such a case, the probability $P(i \to n)$ should be interpreted as the probability of transition per final state and the summation should be changed to an integral over the final states.

The total probability in Equation 5.12.1 requires a sum over the integers cooresponding to the final states $|n\rangle$. Apparently, we imagine the electron lodges itself in one of the final energy basis states. However, we know that the final wave function might also be a linear combination of the energy basis states $|n\rangle$. In such a case, the electron simultaneously exists in two or more states $|n\rangle$ (consider two for simplicity). According to classical probability theory, we must subtract this probability from Equation 5.12.1 to find

$$\text{Prob(A or B)} = \text{Prob(A)}+\text{Prob(B)} - \text{Prob(A and B)}$$

However, we assume that a measurement of the energy of the electron has taken place, the wavefunction has collapsed, and that the electron resides in one of the energy basis states. Therefore the Prob(A or B) reduces to the sum of probabilities as in Equation 5.12.1. Fermi's Golden rule therefore integrates over the range of final states find the number of transitions occuring per unit time.

This section also shows how Fermi's golden rule can be used to demonstrate the semiconductor gain. A detailed treatment must wait for discussions on the denisty operator, the Bloch wave function and the reduced density of states.

Topic 5.12.1: Definition of the Density of States

In this topic, we discuss the counting procedure for the energy density of states. The localized states provide the simplest starting point because we do not need the added complexity of the allowed wave vectors.

The energy density of states (DOS) function measures the number of energy states in each unit energy interval in each unit volume of the crystal

$$g(E) = \frac{\#\text{states}}{\text{Energy} * \text{XalVol}} \qquad (5.12.2)$$

We need to explore the reasons for dividing by the energy and the crystal volume.

First we discuss the reason for the "per unit energy". Suppose we have a system with the energy levels shown on the left side of Figure 5.12.2. Assume for now that the states occur in a unit volume of material (say 1 cm^3). The figure shows 4 energy states in the energy interval between 3 and 4 eV. The density of states at E=3.5 must be

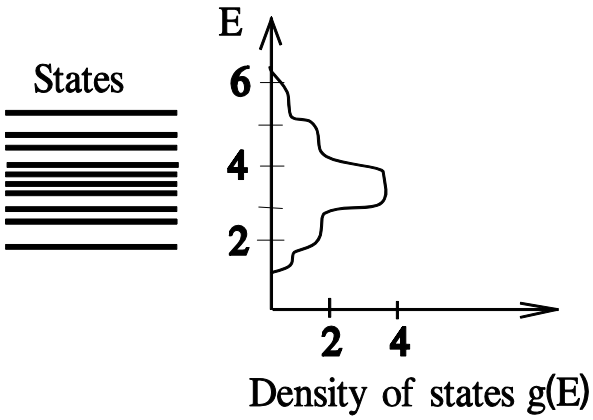

Figure 5.12.2: The density of states for the discrete levels shown on the left-hand side. The plot assumes the system has unit volume (1 cm^3) and the levels have energy measured in eV.

$$g(3.5) = \frac{\#\text{states}}{\text{Energy} \times \text{Vol}} = \frac{4}{1\text{eV} \times 1\text{cm}^3} = 4$$

Similarly, between four and five electron volts, we find two states and the density of states function has the value $g(4.5) = 2$ and so on. Essentially, we just add up the number of states with a given energy. The graph shows the number of states versus energy; for illustration, the graph has been flipped over on its side. Generally we use finer energy scales and the material has larger numbers of states (10^{17}) so that the graph generally appears much smoother than the one in Figure 5.12.2 since the energy levels essentially form a continuum. The "per unit energy" characterizes the type of state and the type of material.

The definition of density of states uses "per unit crystal volume" in order to remove geometrical considerations from the measure of the type of state. Obviously, if each unit volume has N_v states (electron traps for example) given by

$$N_v = \int_0^\infty dE \; g(E) = \int d(\text{energy}) \frac{\#\text{states}}{\text{Energy} * \text{vol}} = \frac{\#\text{states}}{\text{vol}} \qquad (5.12.3)$$

then the volume V must have $N = N_v V$ states. Changing the volume changes the total number. To obtain a measure of the "type of state", we need to remove the trivial dependence on crystal volume.

What are the states? The states can be those in an atom. The states can also be traps that an electron momentarily occupies until being released back into the conduction band. The states might be recombination centers that electrons enter where they

recombine with holes. Traps and recombination centers can be produced by defects in the crystal. Surface states occur on the surface of semiconductors as an inevitable consequence of the interrupted crystal structure. The density of defects can be low within the interior of the semiconductor and high near the surface; as a result, the density of states can depend on position. Later we discuss the extended states in a semiconductor .

Let's consider several examples for the density of states. First, suppose a crystal has two discrete states (i.e. single states) in each unit volume of crystal. Figure 5.12.3 shows the two states on the left side of the graph. The density-of-state function consists of two Dirac delta functions of the form

$$g(E) = \delta(E - E_1) + \delta(E - E_2)$$

Integrating over energy gives the number of states in each unit volume

$$N_v = \int_0^\infty dE\, g(E) = \int_0^\infty dE\, \left[\delta(E - E_1) + \delta(E - E_2)\right] = 2$$

If the crystal has the size 1x4 cm^3 then the total number of states in the entire crystal must given by

$$N = \int_0^4 dV\, N_v = 8$$

as illustrated in Figure 5.12.4. Although this example shows a uniform distribution of states within the volume V, the number of states per unit volume N_v can depend on the position within the crystal. For example, the growth conditions of the crystal can vary or perhaps the surface becomes damaged after growth.

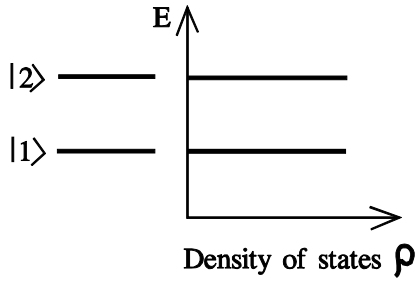

Figure 5.12.3: The density of states for two discrete states shown on the left side.

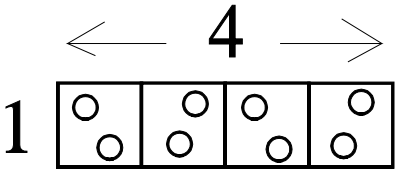

Figure 5.12.4: Each unit volume has two states and the full volume has 8.

As a second example, consider localized states near the conduction band of a semiconductor as might occur for amorphous silicon. Figure 7.11.4 shows a sequence of graphs. The first graph shows the distribution of states versus the position "x" within the semiconductor. Notice that the states come closer together (in energy) near the conduction band edge. As a note, amorphous materials have mobility edges rather than band edges. The second graph shows the density of states function versus energy. A sharp Gaussian spike represents the number of states at each energy. At 7 electron volts, the material has six states (traps) per unit length in the semiconductor as shown in the first graph. The second graph shows a spike at seven electron volts. Actual amorphous silicon has very large numbers of traps near the upper mobility edge and they form a continuum as represented in the third graph. This example

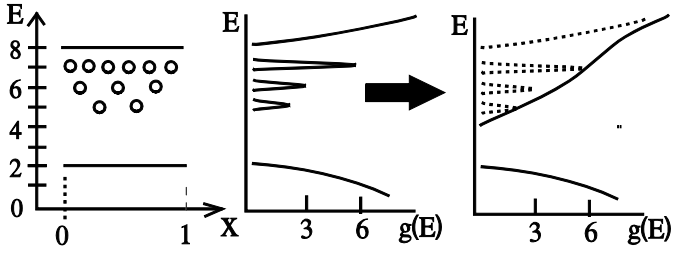

Figure 5.12.5: Transition from discrete localized states to the continuum.

shows how the density of states depends on position and how closely space discrete levels form a continuum.

Topic 5.12.2: Equations for Fermi's Golden Rule

The previous section shows that the probability of a transition from an initial state $|i\rangle$ to a final state $|n\rangle$ can be written as

$$\text{Prob}(i \to n) = |\beta_n|^2 = \left(\frac{\mu_{ni} E_o}{\hbar}\right)^2 \frac{\sin^2\left[\frac{1}{2}(\omega_{ni} - \omega)t\right]}{(\omega_{ni} - \omega)^2} \tag{5.12.4}$$

with an applied electric field of

$$\vec{E}(x,t) = E_o \text{Cos}(\omega t) \tag{5.12.5}$$

which leads to the perturbing interaction energy

$$\hat{V}(x,t) = \hat{\mu}(x)\frac{E_o}{2}\left(e^{-i\omega t} + e^{+i\omega t}\right) = \hat{\mu}(x) E_o \cos(\omega t) \tag{5.12.6}$$

The dipole moment operator $\hat{\mu}$ provides the matrix elements μ_{ni} that describe the interaction strength between the field and the atom. The dipole matrix element μ_{ni} can be zero for certain final states $|n\rangle$ and Equation 5.12.4 then shows that the transition from the initial to the proposed final state cannot occur. As in Section 5.8, the symbol ω_{ni} represents the difference in energy between the final state $|n\rangle$ and initial state $|i\rangle$

$$\omega_{ni} = \frac{E_n - E_i}{\hbar}$$

where ω_{ni} gives the angular frequency of emitted/absorbed light when the system makes a transition from state $|i\rangle$ to state $|n\rangle$. The incident electromagnetic field has angular frequency ω. Equation 5.12.4 gives the probability of transition for each *final* state $|n\rangle$ and each *initial* state $|i\rangle$. In this topic, we are interested in the density of final states but not in the density of initial states. We therefore take the units for Equation 5.12.4 as the *probability per final state*.

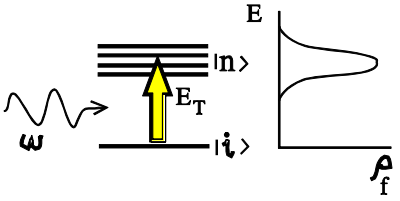

Figure 5.12.6: An electromagnetic wave induces a transition from state "i" to one of the final states.

Equation 5.12.1 shows that the total probability of the electron leaving an initial state "i" must be related to the probability that it makes a transition into any number of final states. How can we change the formula if the final states have the same energy? As an answer, transition to final states all having the same energy must have equal probability as can be seen from Equation 5.12.2 (the same ω_{ni}). For N final states with the same energy, we then expect

$$\text{Total Prob} = \sum_n P(i \to n) = N\,P(i \to n)$$

What is the transition probability if some of the final states have energy E_1, some have energy E_2 and so on? Let $\rho(E_n)$ be the number of states at energy E_n (i.e., in the continuum limit, ρ denotes the density of states). Then we expect

$$\text{Total Prob} = \sum_n P(i \to n) = \rho(E_1)P(i \to 1) + \rho(E_2)P(i \to 2) + ... = \sum_n \rho(E_n)P(i \to n)$$

Therefore, for a unit volume of crystal, the *total* probability of transition P_V can be written as

$$P_V = \sum_E \left(\frac{\#\ \text{states}}{\text{energy}\ \ \text{vol}}\right)\left(\frac{\text{prob}}{\text{state}}\right) \Delta E \to \int dE\, \rho(E) P(i \to n) \qquad (5.12.7)$$

where $P(i \to n) = P(E_i \to E_n)$ is the probability of transition (per state) and the integral must be over the energy of the final states. Inserting Equation 5.12.4 into Equation 5.12.7 to find

$$P_V = \int dE\, \rho(E) \left(\frac{\mu_{ni} E_o}{\hbar}\right)^2 \frac{\sin^2\left[\frac{1}{2}(\omega_{ni} - \omega)t\right]}{(\omega_{ni} - \omega)^2}$$

where the transition frequency

$$\omega_{ni} = (E_n - E_i)/\hbar = (E - E_i)/\hbar$$

includes the energy of final states E. It is more convenient to write the integral in terms of the *transition* energy

$$E_T = E - E_i = \hbar\omega_{ni}$$

E_T, which is the energy between the initial state and final states as shown in Figure 5.12.6. We find

$$P_V = \int dE_T\ \rho(E_i + E_T)\ (\mu_{ni}\, E_o)^2\ \frac{\sin^2\left[\frac{1}{2\hbar}(E_T - \hbar\omega)t\right]}{(E_T - \hbar\omega)^2} \qquad (5.12.8)$$

The quantity $\hbar\omega$ represents the energy of the electromagnetic wave inducing the transition. The dipole matrix element μ_{ni} depends on the energy of the final state E through the index "n". Therefore the dipole moment can be written as $\mu_{ni} = \mu(E)$ for fixed initial state i. In this section, we assume that the dipole matrix element to be independent of the energy of the final state. Therefore we take $\mu_{ni} = \mu$ to be a constant and remove it from the integral in Equation 5.12.8. This assumes that the final states all have the same transition characteristics; the interaction strength between the electromagnetic wave and the system (i.e., atom) remains the same for all possible final states under consideration.

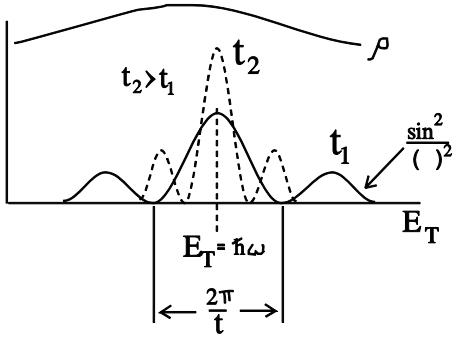

Figure 5.12.7: The "S" function becomes very narrow for larger times.

Next, look at the last term in the integral in Equation 5.12.8

The Livermore experience: Contributions of J. H. Eberly to laser excitation theory

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Abstract: This article summarizes the developing understanding of coherent atomic excitation, as gained through a collaboration of J. H. Eberly with the Laser Isotope Separation Program of the Lawrence Livermore National Laboratory, particularly aspects of coherence, population trapping, multilevel multiphoton excitation sequences, analytic solutions to multistate excitation chains, the quasicontinuum, pulse propagation, and noise. In addition to the discovery of several curious and unexpected properties of coherent excitation, mentioned here, the collaboration provided an excellent example of unexpected benefits from investment into basic research.

OCIS codes: (270..1670) Coherent optical effects

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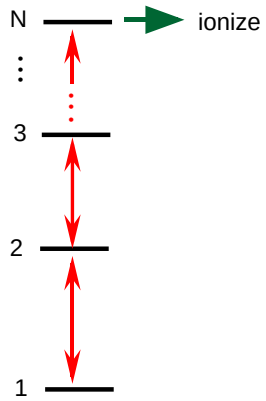

Fig. 1. Schematic diagram of resonant laser excitation of a chain of N energy levels, followed by ionization. Excitation energy increases upwards. Vertical red arrows indicate connections induced by various lasers. Horizontal arrow indicates (ionization) probability loss.

privatized as a part of the United States Enrichment Corporation (USEC). That program ended only after a decision in 1998 by top management of the USEC to cancel totally the support of research and development of this technology. The failure to deploy AVLIS for civilian use was in part a consequence of the very limited market growth in demand for nuclear fuel in the last two decades as well as the very large supply made available (through blending) from weapons grade stockpiles after the Cold War ended; it was not a failure of the science or technology.

The concept pursued at Livermore, with little deviation, was in essence the following simplified process. Start with solid chunks of ordinary uranium, melt and vaporize it under vacuum, form a beam of atomic vapor, and expose the streaming vapor to several coincident beams of laser light. The laser frequencies were carefully chosen to match Bohr transition frequencies along an excitation chain of increasingly more energetic bound states, eventually terminating with an autoionizing state embedded in the photoionization continuum; see Fig. 1. The result of the laser exposure was to photoionize only a chosen isotope (because other isotopes would not be resonant with the lasers). Electrostatic fields would then separate the ions (the desired isotopes) from the background of neutral atoms (the undesired isotopes.)

Very early in the project it was recognized that, in addition to engineering and materials handling challenges, there were many questions of a very fundamental nature that needed to be addressed in order to place the modeling of the separation process on a secure foundation. Indeed, a group of theorists with expertise in chemistry and physics issues (the Theoretical Atomic and Molecular Physics group, or TAMP, headed by Charlie Bender) were assembled, in part to address these. During the startup of the LIS project it was headed by Ben Snavely, who came to Livermore from Eastman Kodak in Rochester. There he had been acquainted with Joe Eberly, and knowing of Joe's ability to ask and answer very fundamental questions about laser excitation, Ben hired Joe as a consultant. Joe's scientific points of contact at LLNL were Bruce Shore and Mike Johnson. Almost from the beginning of this consultantship, which typically involved two visits a year to Livermore, the LIS project made funds available to support basic research at Rochester. You have to understand that even two decades ago the organization of our national laboratories was very different than it is now. Places such as LLNL were able to make funding grants for basic research at the discretion of program leaders, and under the enlightened leadership of Ben Snavely and his successor Jim Davis (1974 to 1986), there was ongoing support for post-docs and students at Rochester. Indeed,

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Joe recently pointed out to BWS that his most widely cited reference [12] was financed with support from LLNL. In that paper they predicted and explained the "collapse and revival" of population oscillations of a two-state atom in a single-mode cavity, one of the few definite ways in which the discreteness of photons is observable. Though unrelated to any application at LLNL, that paper has had a significant impact on all of quantum optics. Sadly, a change of management at Livermore brought an abrupt and unforeseen termination to this work, and the consultantship, in 1987. This document reviews some of the things we learned during this collaboration, doing research that formed the core of a major treatise on coherent excitation, where more details can be found [8]. In the words of our honoree,

> *"Recall those wonderful days in Livermore when we knew only epsilon more than anybody else there, but epsilon was enough."*

Unfortunately it is not possible, in the short space available here, to present a comprehensive discussion of the topics in this article. We aim primarily to summarize work at LLNL on coherent excitation, all of which was influenced by Joe. We have cited all of the joint publications with Joe and LLNL. For further details, and many more references to related and historically significant works, readers should consult the aforementioned book [8].

2 Beginnings

The concept of selective multistep photoionization that formed the basis for the LIS scheme at LLNL was very simple: one chose a set of laser frequencies that would provide a resonant excitation chain from the ground state into the photoionization continuum. The precise wavelengths would be determined by experiment (and would be held in secrecy). The basic challenge for theory was to predict the intensity of the various lasers, given the measured oscillator strengths, such that the ionization would proceed selectively and approach completeness asymptotically - all at least cost.

At that time theorists dealing with radiation effects on vapors gained their understanding from studying textbooks aimed at astrophysicists who sought to model the passage of radiation through stars. The relevant equations expressed the rate of change in atomic populations as being proportional to the energy density (or the flux) of radiation [8]. The proportionality coefficients were the Einstein-Milne B coefficients (or cross sections). It was these radiative rate equations that were used in the first modeling of laser excitation in the LIS program by Rich Davis.

However, even undergraduate physics majors at that time had encountered the time-dependent Schrödinger equation, and knew it as the basic equation governing time evolution at the most fundamental atomic level. This equation differed very significantly from the Einstein rate equations: Rather than deal with linear differential equations for probabilities, it dealt with differential equations for probability amplitudes. Only after squaring these amplitudes did one obtain the observable probabilities. The reliance on amplitudes leads to the possibility of both constructive and destructive interference effects, and so it is possible to obtain very different results from the two approaches.

Surprisingly little had been done with the time-dependent Schrödinger equation at that time. Apart from some special cases mentioned below, it was regarded primarily as a means of deriving rate coefficients by means of time-dependent perturbation theory and Fermi's famous golden rule.

One of the first fundamental questions that had to be addressed when considering laser-induced atomic excitation was: what equations would describe the time evolution of an illuminated vapor, as it would be used in the LIS project? Rate equations or the

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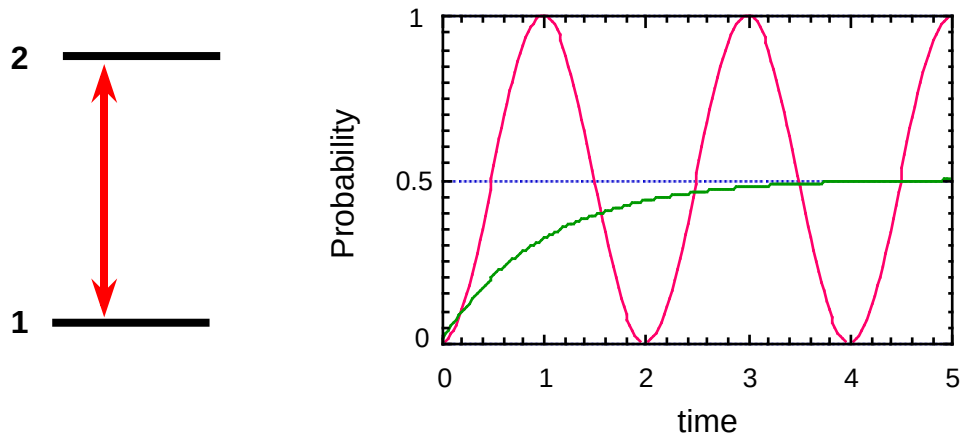

Fig. 2. Time dependence of resonant excitation probability $P_e(t)$ for lossless two-level atom. Monotonic green curve is for rate equations, oscillatory red curve is for the time-dependent Schrödinger equation (oscillation frequency is the Rabi frequency).

Schrödinger equation? Stated somewhat differently, were we to deal with (incoherent) multiple photon absorption or (coherent) multiphoton absorption? As we now understand from numerous textbooks on quantum optics and laser physics, these two types of equations are extreme cases of a formalism that can be dealt with by means of density matrices [8]. When excitation occurs by means of coherent radiation (laser light) then the Schrödinger equation comes close to the correct description. With incoherent light (the astronomical sources or plasma sources) then rate equations are suitable. But this was not so clear in those early days.

In one of his early visits to the LIS project, Joe participated in a lively discussion of the significance of coherence for LIS, organized at the suggestion of Jim Davis, who professed skepticism about the need to consider coherence (and some of the curiosities of the Schrödinger equation) in any practical separation program. The disputants at that time included, besides Joe, Bruce Shore, John Garrison, Mike Johnson, and a few others. Joe gave a masterful lecture on the two-level atom, starting from the most basic ideas of probabilities and the Schrödinger equation, going through what is now very traditional introduction of the rotating wave approximation (RWA), and ending with sinusoidal Rabi oscillations of populations. (These contrast with the monotonic growth of populations illuminated incoherently, as predicted by rate equations; see Fig. 2). All of this was still new and novel at that time. Davis was unconvinced, however, and on the spur of the moment offered a challenge, to be known as The Davis Cup, to anyone who could convince him that coherence was important in his job as leader of the LIS project.

Eventually, largely as the result of several years of collaboration between Joe and Bruce, Davis acknowledged that it was indeed important to base modeling on the Schrödinger equation rather than rate equations, and he graciously made an award of The Davis Cup (to BWS). The original cup was simply a styrofoam coffee cup (probably the one used by Davis himself that day), but eventually it became a heavy vessel of machined brass, mounted on a mahogany base.

3 The Excitation Chain

Already in 1976 Joe had wondered about a very basic issue concerning a chain of excitations, such as those indicated in Fig. 1. It was known that, in a two-state excitation

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followed by ionization, a sufficiently high ionization rate would damp out the Rabi oscillations and give results that were predictable from rate equations. What would happen if there were a chain of excitations, leading to a final ionization? Would an increase of the ionization rate cause the entire chain to lose characteristics of coherent excitation? Or would the incoherence be confined to the final step?

With his then graduate student Jay Ackerhalt, Joe answered these questions as follows: incoherence affects only the final stage at first, but as that stage becomes incoherent, then it can affect the preceding stage [13]. Ultimately one can have a completely incoherent sequence, in which the excitation rates proceed faster and faster as the population rises along the excitation sequence. Interesting though this regime is, it turned out not to be an optimum for purposes of isotope separation.

4 Jay Ackerhalt

One of the early benefits to Livermore from the collaboration with Joe was the arrival at LLNL of Jay Ackerhalt in 1976, fresh from his PhD work at Rochester where he had been Joe's first graduate student, and had devised an elegant way of treating spontaneous emission by means of a source field and Heisenberg equations of motion. Jay was only briefly at LLNL, before moving to his career at Los Alamos, but his work at Livermore, including his code BICENT, helped elucidate the connections between rate equations and the Schrödinger equation [13, 14, 15, 16]. Though his stay at LLNL was brief, he participated in many enjoyable discussions at Livermore. Joe and Bruce were amongst the speakers at the special memorial session for Jay held in September 1992 at the Institute of Laser Science Convention.

5 The Three State Atom

Although our first theoretical concerns were with two-level atoms (Joe was, after all, renown for co- authoring with Les Allen the classic textbook on two-level atoms [17]), very soon we began considering the next logical extension, the three-level atom. It will seem quite curious to readers today, but at that time the three-level atom had not been subject to very much scrutiny (examples of other work include [18, 19], see [8]), and we published a paper in which we described some of the most elementary properties of the three-state system, subject to steady radiation fields [15]. In particular, we presented analytic solutions for the probability amplitudes, something that may seem obvious in retrospect but was, at the time, still publishable. This analysis based on the Schrödinger equation, and its implied complete coherence, was followed by an elucidation of the changes in excitation that would be produced by incoherence, as described by a density matrix [14].

Although this step, from two states to three, was a significant advance at that time, we had no idea of the remarkable effects that would much later be discovered when sequential pulses act on a three-level atom. The effects of counter-intuitive pulse sequences came to be recognized only after work at Rochester involving Fuk Hioe, Yossi Oreg and Joe [20].

6 The Lambda System: Dark States

One of the most remarkable novelties of the three state atom became obvious during our numerical modeling of three-state excitation. Suppose you have a two state atom, resonantly excited by a steady beam of radiation. Suppose further that the excited state can ionize, perhaps by an additional steady photoionizing field. Then a long steady pulse will eventually completely deplete the initial state, converting all the atoms into ions. This is pretty obvious, though there are some subtleties that may not be obvious at first.

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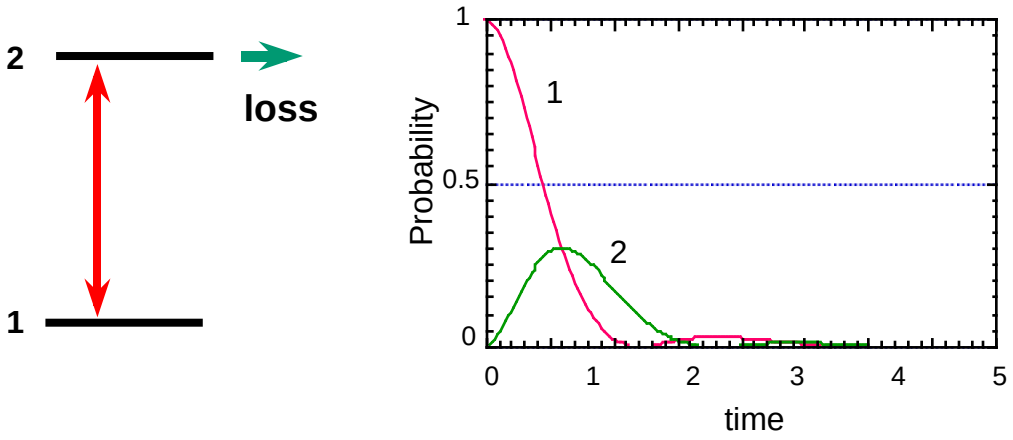

Fig. 3. Time dependence of resonant excitation probabilities $P_n(t)$ for two-level atom with loss from upper level. Curves are marked with level number n. Times are in units of the loss rate; the Rabi frequency is twice this rate. The diagram at the left shows the laser-induced excitation linkage between the two levels. All population is eventually lost.

(For example, if the photoionizing radiation is made very intense, it will actually slow the rate of ionization.) What is quite unexpected is that if you have a second low-lying state, initially populated, and you link this state with the same ionizing excited state by means of a second resonantly tuned laser field, you will not obtain complete ionization. No matter how intense the two excitation fields, and how long you wait, some population will remain in the two low-lying states. Figures 3 and 4 illustrates the dramatic change produced by adding a second leg to the excitation linkage, in the so-called "lambda" configuration.

Nowadays it is understood that this un-ionized population is trapped in a coherent superposition state, a so-called "dark state" or "population trapping state" [21]. But our first encounter of this phenomena was quite unexpected [22]. Carlos Stroud subsequently pointed out to BWS that this coherence had been discovered not only during work with his students Rich Whitley and Bob Gray [23, 24] but some years before, by Arimondo and Orriols [25] who nowadays get the credit for observing this population trapping effect in optical transitions. A rather simple example of exactly this coherent effect is to be found in the example of coordinate choices for treating excitation involving degenerate magnetic sublevels, identified by magnetic quantum number M, of transitions between states of well defined angular momentum J. Figure 5 illustrates this.

Population trapping states are an essential prerequisite for the success of various schemes for transferring population adiabatically, as in the Stimulated Raman Adiabatic Passage (STIRAP) process [26, 27]. The foundation for this line of work was laid at Rochester, and described in a paper by Eberly, Hioe and Oreg [20] who pointed out how adiabatic states (i.e. instantaneous eigenstates of the Hamiltonian), in multilevel systems, can be used to carry population between specified physical states by means of suitably crafted laser pulses. The significance of this theoretical work became evident with the experimental work of Klaas Bergmann and his co-workers; for a review see [28, 29]. Here too, significant questioning by Joe brought new insights into this process [30].

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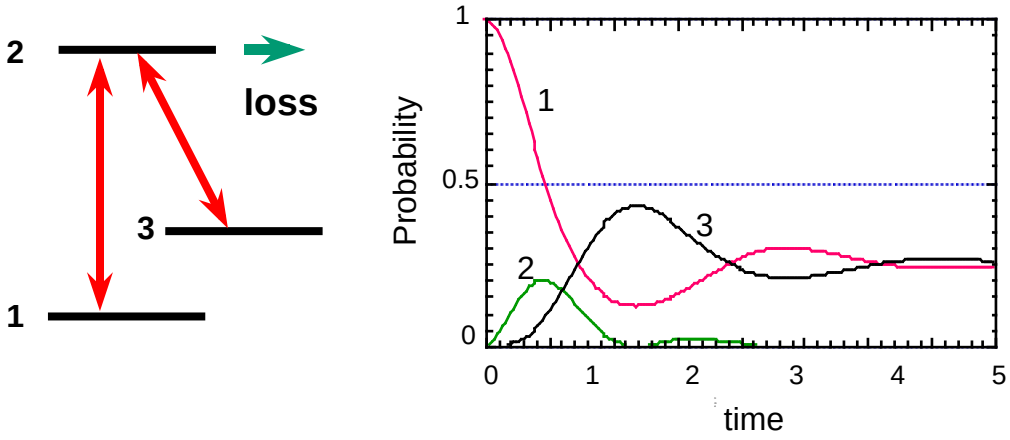

Fig. 4. Time dependence of resonant excitation probabilities $P_n(t)$ for three-level lambda system, with loss from level 2, for population initially all in level 1. Curves are marked with level number n. Times are in units of the loss rate; the Rabi frequencies are each twice this rate. The diagram at the left shows the laser-induced excitation linkages between the three levels. After a long time one fourth the population resides in level 1 and another fourth in level 3; only half has been lost.

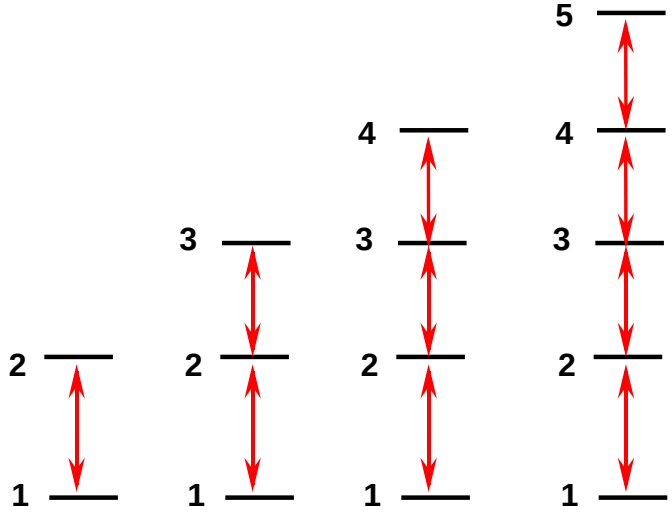

Fig. 5. Example of linkages of linearly polarized light between magnetic sublevels of a transition between angular momentum $J = 1$ and $J = 0$. (a) Using a coordinate system in which the z (quantization) axis lies along the electric field. (b) Using a coordinate system in which the electric field direction is taken as the x axis, and the light is considered a coherent superposition of right- and left-circular polarization. This is an example of the lambda system of Fig. 4; it is equivalent to the linkage of (a).

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around 1987. But this did not end his Livermore connection, it only redirected the points of contact.

To place this new connection into context one needs to recall the work by Joe and his student Zhifang Deng (son of the then Premier of China), aimed at providing a simple understanding of some of the properties of a photoionization continuum then being experimentally discovered [56, 57, 58]. For years most physicists had regarded the electronic states above the ionization limit as an incoherent sink of probabilities. The view was that an electron, once ejected from an atom, was forever lost. But experiments demonstrated that an electron, in leaving the atom under the influence of a strong laser field, could absorb more than the minimum number of photons needed to overcome the binding energy. These excess photons produce a succession of peaks in the photoelectron spectrum, a phenomena that became known as above threshold ionization (ATI) [59]. During a visit to Livermore, Peter Knight recognized that a structured continuum offered an opportunity to enhance the production of harmonics of the strong laser field [60], as subsequently was demonstrated experimentally. Prompted in part by the development of laser sources capable of producing brief electric fields that would overwhelm the binding field of the nuclear attraction on electrons, theorists were examining a new regime of atomic and optical physics. Much of this theoretical work made use of techniques for modeling an electron in space and time, an area in which Ken Kulander at Livermore was uniquely qualified to contribute [61, 62, 63]. This collaboration did much to clarify the complicated processes that occur when atoms are exposed to intense radiation fields [64, 65].

16 Closing Remarks

The work of Joe Eberly for Livermore from 1973 to 1987 not only helped establish the basic conditions needed for successful commercial laser-induced isotope separation, but it also revealed many of the interesting properties of coherent atomic excitation [8]. It is the latter aspect of his collaboration, documented in more than a dozen papers, that holds the more lasting legacy for science. The collaboration came at a time when it was still considered desirable that a National Laboratory engage in and support not only applied research directed at finding an immediate solution to some identified engineering problem but also basic research intended only to enlarge the base of knowledge in physics.

This basic research at LLNL underlay the ultimate success of the theoretical modeling effort, based on a computer code written by Bob Nelson (and later extended by Ron White) that combined multiphoton ionization of the atoms (described by the time-dependent Schrödinger equation) and propagation of the laser beams (based on the Maxwell equations). Using only experimentally determined oscillator strengths and wavelengths, the theory was confirmed (without free parameters) for optically thick and thin transitions over very long propagation paths. This was a remarkable accomplishment considering the complexity of the theoretical modeling, which included hyperfine structure, polarization effects (magnetic sublevels), stimulated Raman scattering, etc., and the daunting experimental challenges of measuring absolute photoionization yields.

17 Acknowledgments

We are pleased to honor Joe Eberly by recalling fondly his many contributions to science at the Lawrence Livermore National Laboratory; to him we dedicate this review article.

BWS says: The years of collaboration with Joe Eberly have been particularly memorable and valuable for me personally; the many long enlightening discussions with him, and with Peter Knight, formed the basis for what subsequently became a major two-

volume textbook on coherent excitation [8]. One of the great treats of a visit with Joe at Livermore was his skill in placing our various computational efforts into a larger landscape. He would inevitably point out some greater significance to our work than we had recognized. And he was a great dinner guest. My children Tim and Hilary share fond memories of suppertime conversations about Louis XI, the "universal spider", and demonstrations of strange attractors on a simple pocket calculator.

MAJ says: Those were heady times for a new Ph.D.-spending hours around hallway tables "arguing" with Joe about how atoms and photons really behaved, making the connections from two-level atoms and perfect plane waves to the laboratory reality of uranium atoms with their 400 relevant sublevels and not-so-perfect, not-so-monochromatic lasers. The textbooks may indeed have had in them all the theory we required, but Joe's voice carrying down the hall was the excuse we welcomed to show someone the data and figure out what it really meant.

KCK says: Just knowing Joe has had many benefits. Ann Orel and I were trying to get some financial support from a young DOD contract monitor to study a laser-molecule collision process. I was schmoozing with said fellow regarding our proposal and happened to mention that Joe was visiting us. He said, "You know Joe Eberly!?" I said, "Of course." Our funding was awarded in full shortly thereafter.

JID says: I had only recently joined the Lab from industry when I met Joe in 1974. By then, I was a somewhat hardened industrial physicist, and had already developed a healthy skepticism of excessive theorizing about practical problems. I let Joe and his colleagues at the Lab know that they had my support for developing a comprehensive theory of multiphoton ionization of atoms in the vapor phase providing it would be useful to the furtherance of our understanding and progress in developing AVLIS. Joe and Bruce Shore and others accepted this challenge and within a matter of months established the need for a detailed quantum mechanical theory of the multiphoton processes involved. Joe's professional and personal approach were essential in establishing the initial effort and he continued to be a very positive contributor for more than a decade during the entire time that I was director of the AVLIS program. I personally appreciate Joe, not only for what he did for the Lab and AVLIS but especially for his savoir faire.

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